



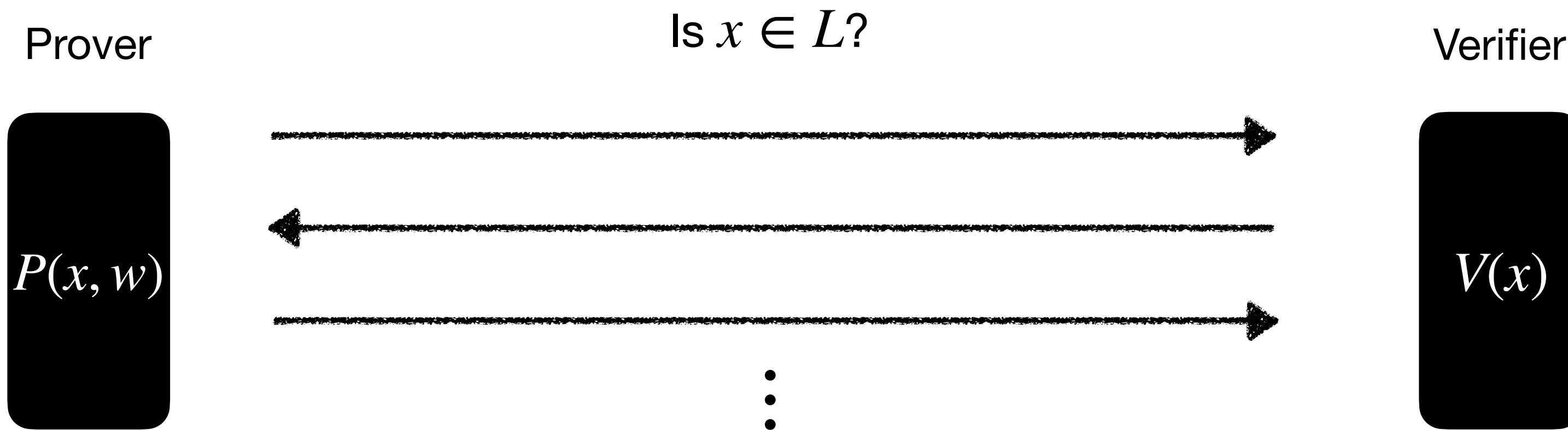
Quantum Rewinding for IOP-Based Succinct Arguments

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Joint work with Alessandro Chiesa, Marcel Dall'Agnol, Zijing Di, and Nick Spooner

What are succinct arguments?

Interactive proofs



Completeness: $\forall x \in L, \Pr [\langle P(x, w), V(x) \rangle = 1] = 1$

Soundness: $\forall x \notin L$ and adversary $\tilde{P}$, $\Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon$

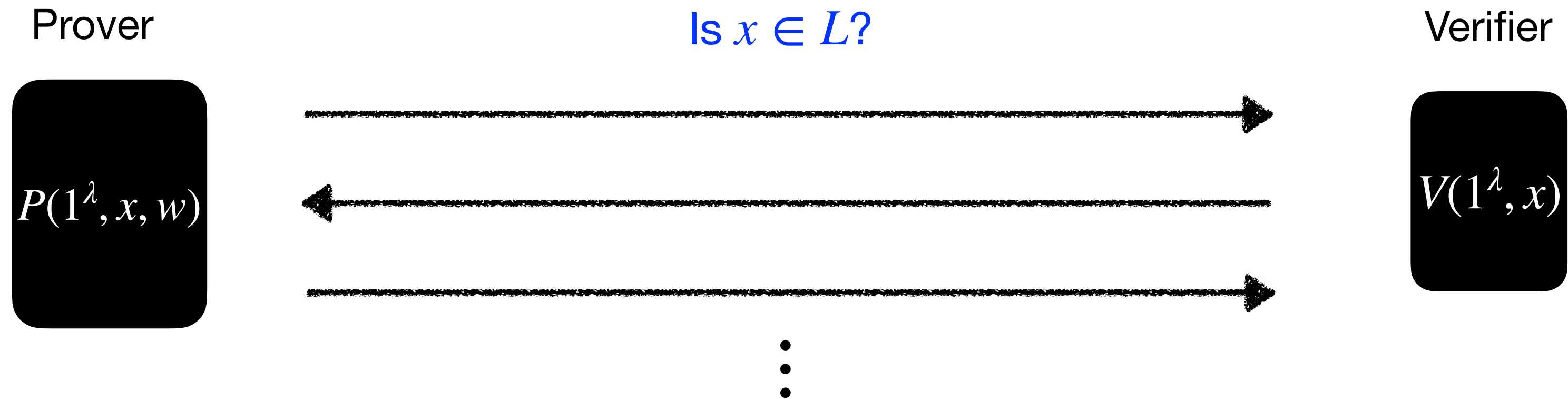
Target metric: **COMMUNICATION COMPLEXITY**

Limitation: NP-complete languages do not have IPs with $\mathbf{CC} \ll |w|$

[GH97]: $\text{IP}[\mathbf{CC}] \subseteq \text{BPTIME}[2^{\mathbf{CC}}]$

Interactive arguments

Interactive proofs with **computational** soundness



Computational soundness: $\forall x \notin L$ and t_{ARG} -time adversary $\tilde{P}$, $\Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon_{\text{ARG}}(t_{\text{ARG}})$

AMAZING: $\exists$ interactive arguments for NP with **CC** $\ll |w|$ (given basic cryptography)

Today's protagonist:
Succinct Interactive Arguments

Why study ^{$cc \ll |w|$} **succinct** ^{$time(V) \ll |w|$} interactive arguments?

They exist based on simple crypto assumptions...

... so they play a role in numerous cryptottheory results.

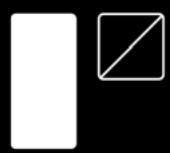
zero-knowledge with
non-black-box simulation

malicious MPC

...

They are a stepping stone for SNARGs, which have numerous real-world applications.

 **Succinct**

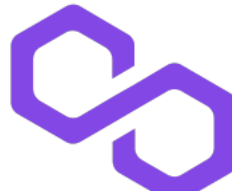
 **RISC
ZERO**

 **Aztec**

 **VALIDA**

Irreducible

 **STARKWARE**

 **polygon**

 **NEXUS**

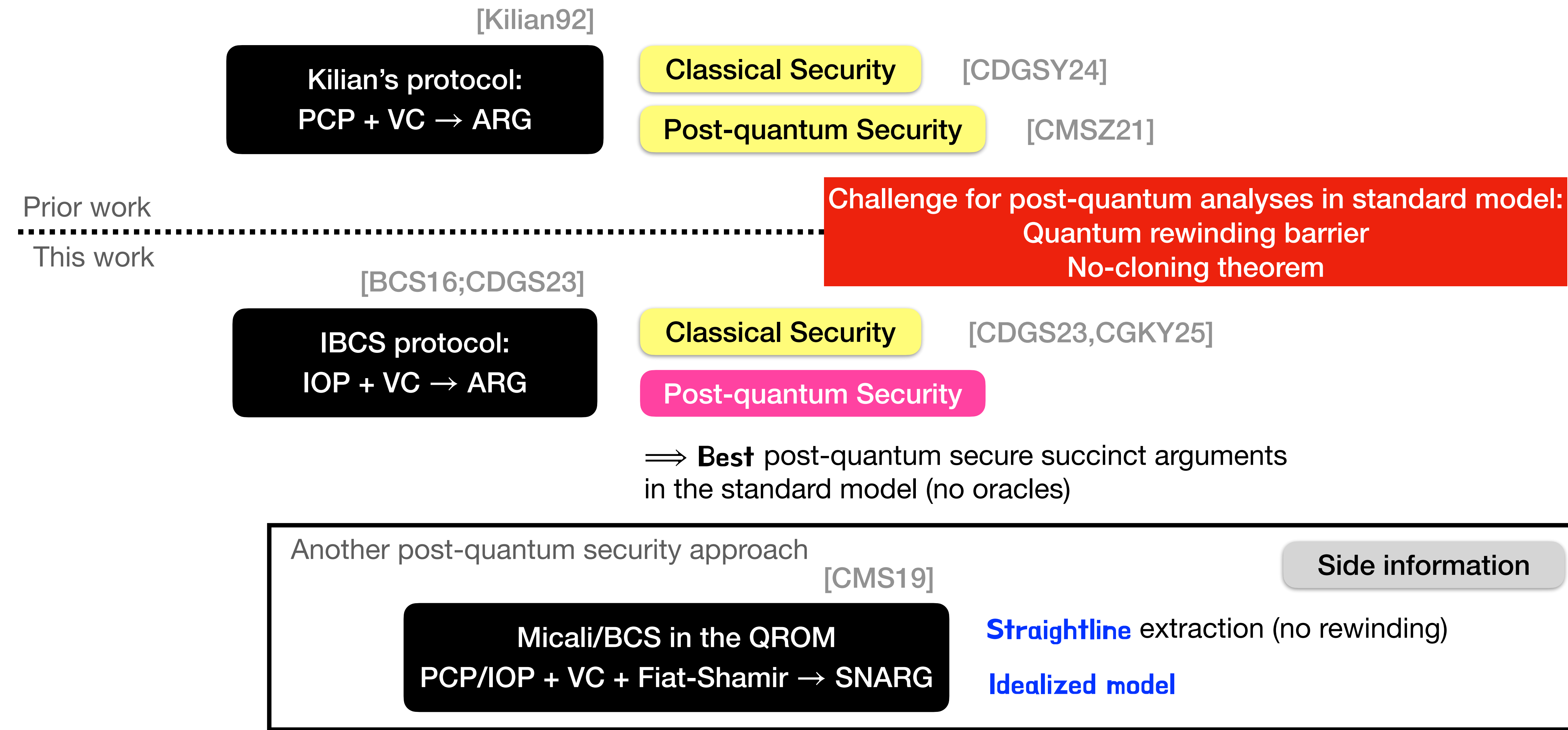
 **Ligero**

 **Matter Labs**

...

Roadmap

Today: only soundness
Knowledge soundness similar

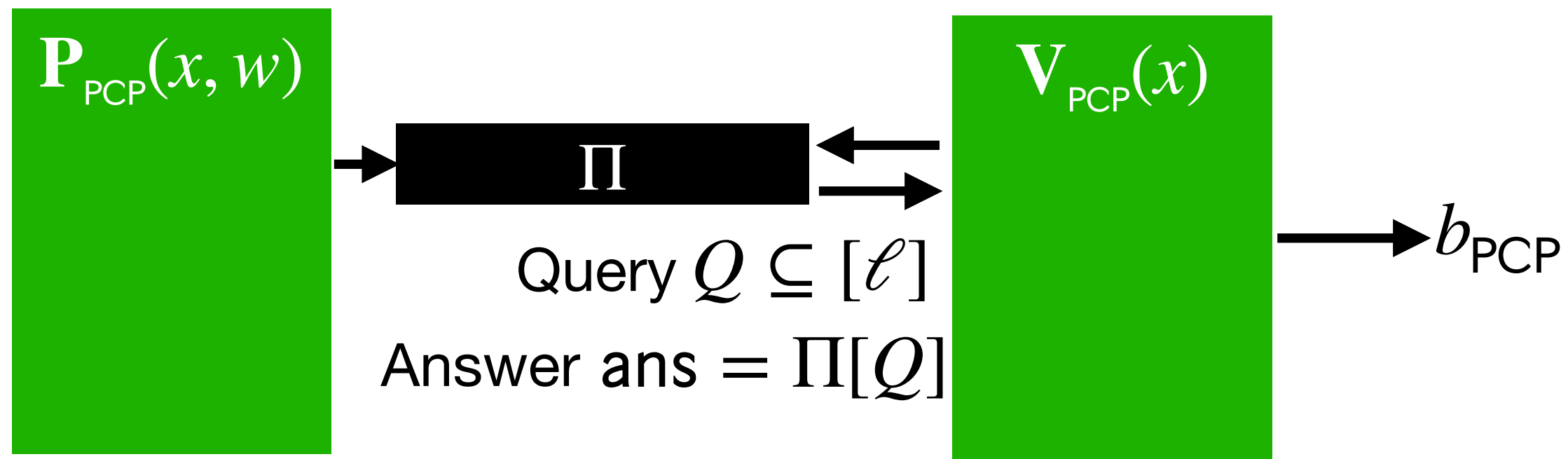


Warm-up: Kilian's protocol

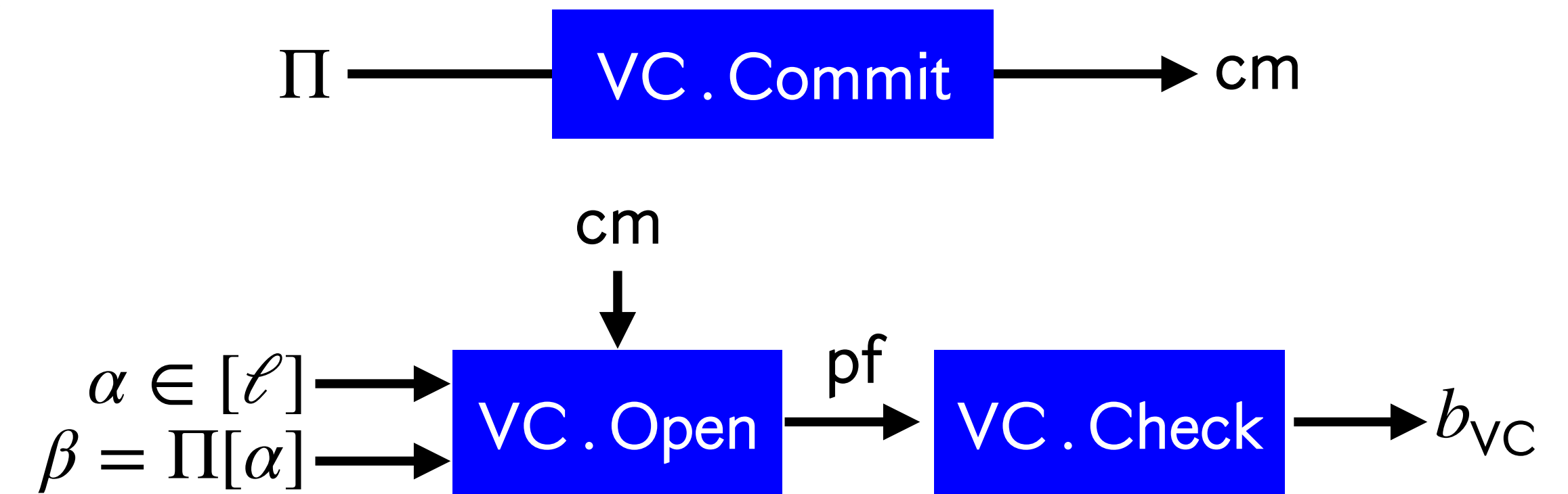
The first and simplest succinct argument

How to construct succinct arguments?

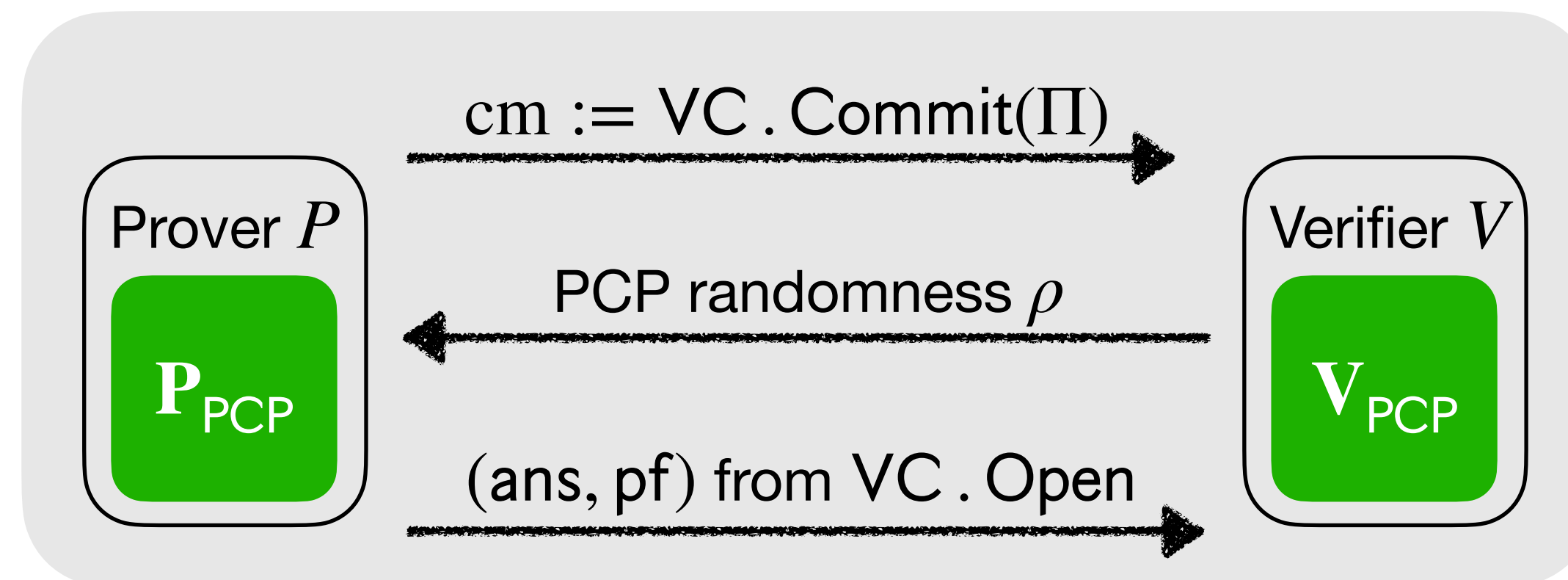
Building block #1: probabilistically checkable proof (PCP)



Building block #2: vector commitment scheme (VC)

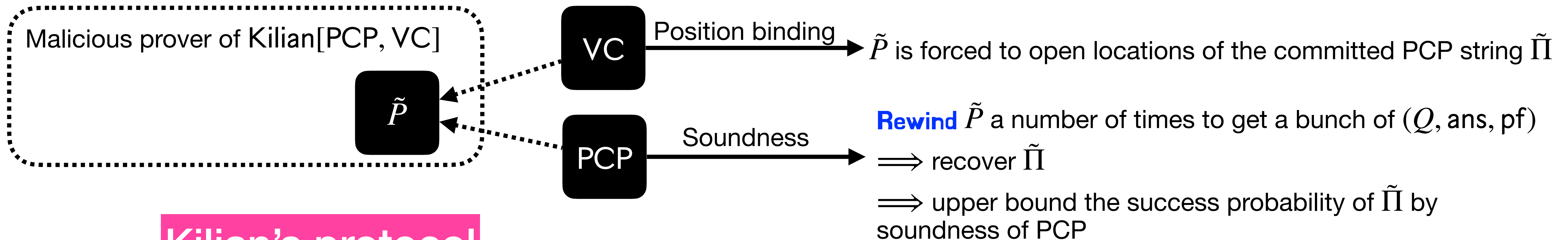


Kilian's protocol

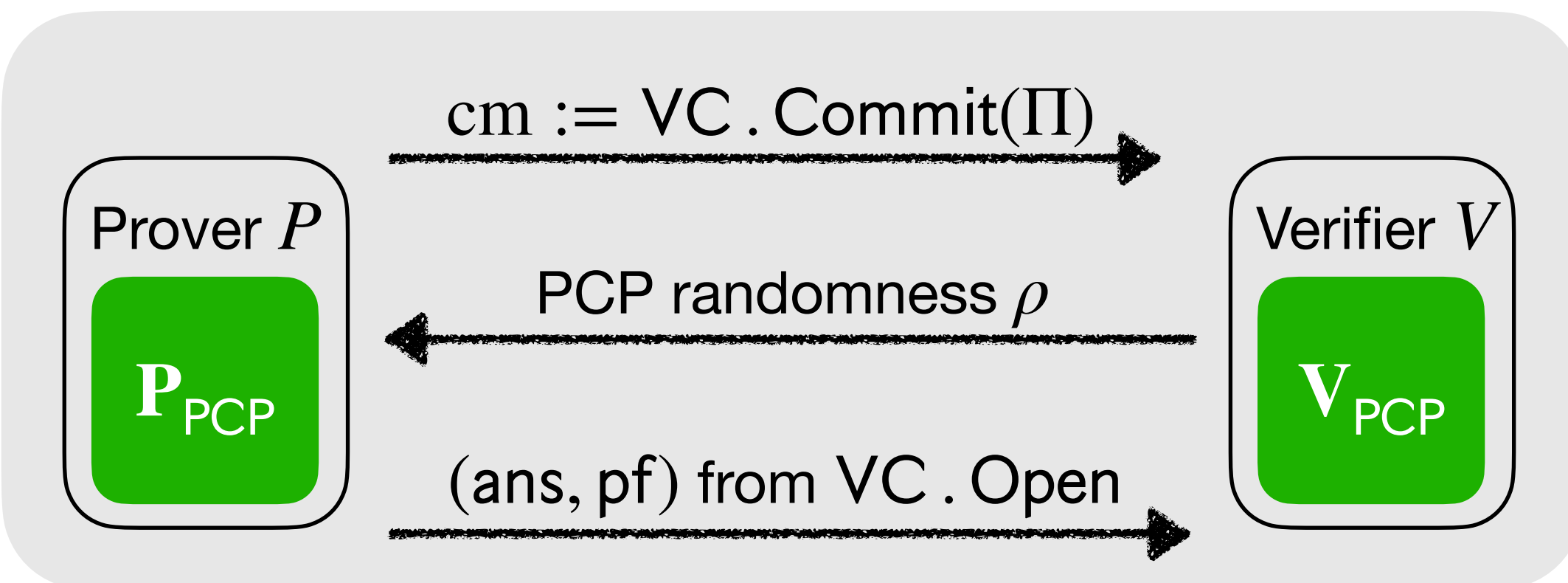


Classical security analysis

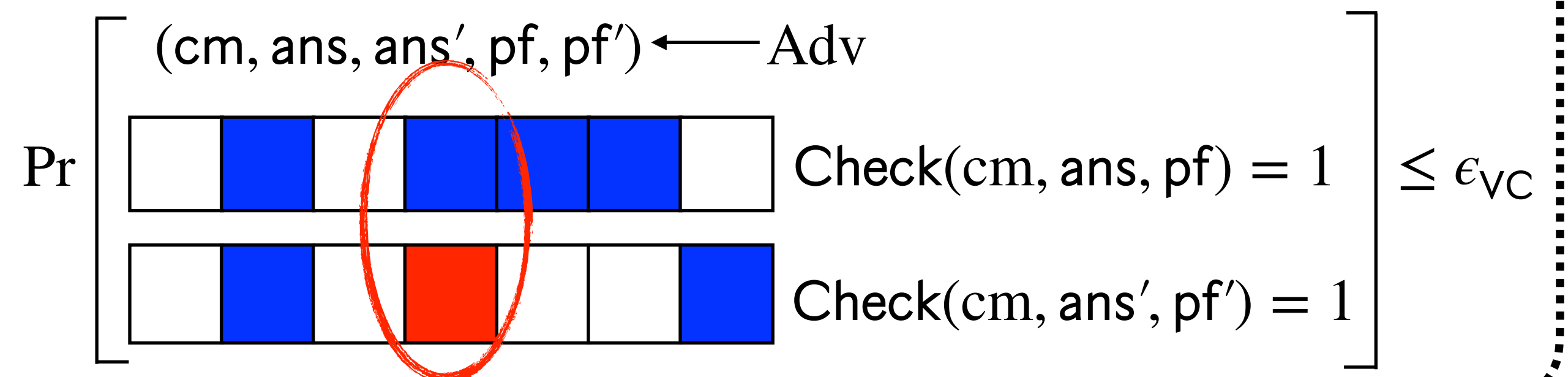
Goal: relate the soundness error of Kilian[PCP, VC]
to the soundness error of PCP and the position binding error of VC.



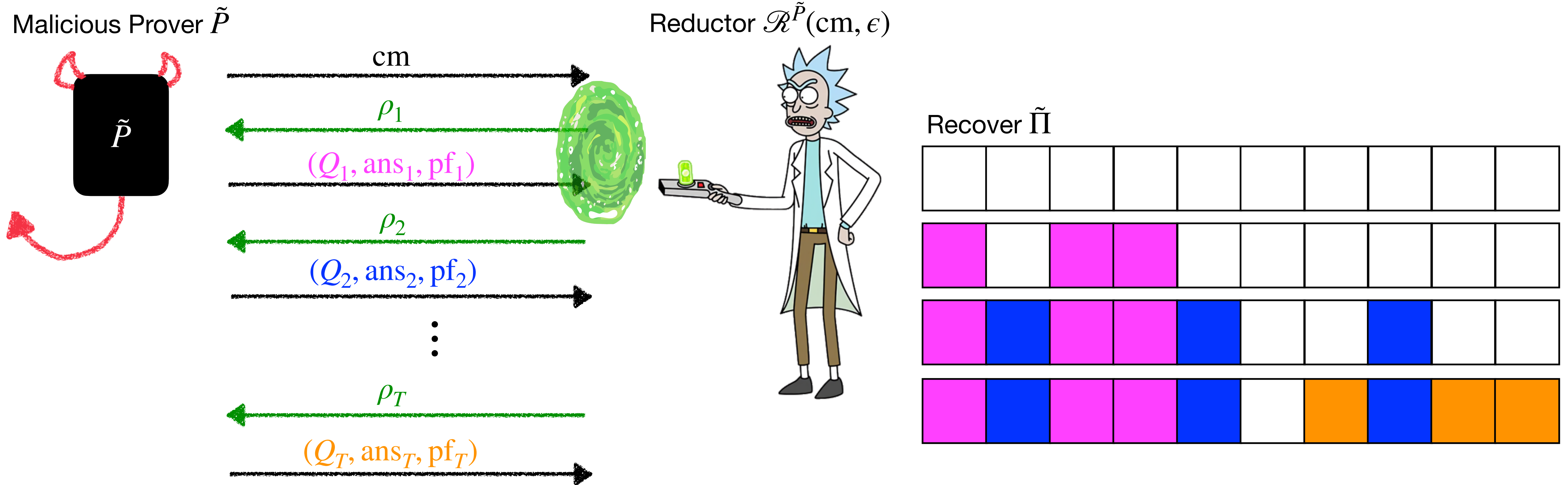
Kilian's protocol



Position binding



Approach: rewind the prover



Theorem [CDGSY24]. $\forall \text{PCP}(\epsilon_{\text{PCP}}, \ell_{\text{PCP}}), \text{VC}(\epsilon_{\text{VC}}), \epsilon > 0,$
 $\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot \ell_{\text{PCP}} \cdot \frac{1}{\epsilon}).$

Overhead from rewinding.
Possibly inherent [CDGSY24]

How about post-quantum security?

Post-quantum soundness: same as classical soundness but adversary is quantum:

$$\forall t_{\text{ARG}}\text{-time QUANTUM adversary } \tilde{P}^*, \Pr [\langle \tilde{P}^*, V \rangle = 1] \leq \epsilon_{\text{ARG}}^*(t_{\text{ARG}})$$

Ethereum Unlocks Millions To Prepare For The Post-quantum Era

Sun 11 May 2025 • 4 min read • by Mikaia A.

The building blocks need to be post-quantum secure at the minimum

PCP for language L with soundness error ϵ_{PCP}

PCP has statistical soundness!

$$\implies \forall x \notin L \text{ and quantum adversary } \tilde{P}, \Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon$$

Vector commitment scheme with position binding error ϵ_{VC}

post-quantum

post-quantum
Position binding

Classical messages

Quantum algorithm

$$\Pr \left[\begin{array}{l} (\text{cm}, \text{ans}, \text{ans}', \text{pf}, \text{pf}') \leftarrow \text{Adv} \\ \begin{array}{|c|c|c|c|c|c|} \hline \text{ } & \text{blue} & \text{ } & \text{blue} & \text{blue} & \text{blue} \\ \hline \end{array} \text{Check}(\text{cm}, \text{ans}, \text{pf}) = 1 \\ \begin{array}{|c|c|c|c|c|c|} \hline \text{ } & \text{blue} & \text{red} & \text{ } & \text{ } & \text{blue} \\ \hline \end{array} \text{Check}(\text{cm}, \text{ans}', \text{pf}') = 1 \end{array} \right] \leq \epsilon_{\text{VC}}$$

Is this sufficient?

Not with current rewinding techniques...

**Key property for rewinding:
Collapsing**

Quantum reductor

Malicious Prover $\tilde{P}^\star$

Quantum algorithms, but output classical messages

$$\tilde{P}^\star = (U_{\text{cm}}, U_{\text{open}})$$

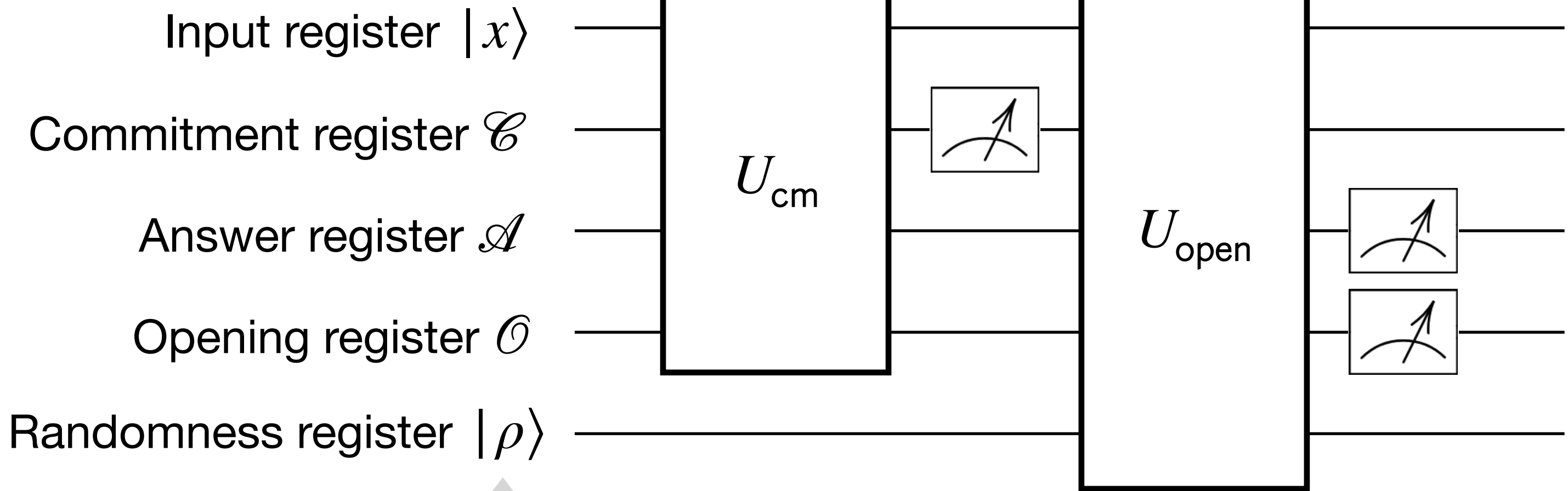


What does it mean to have black-box access to $\tilde{P}^\star$?

Reductor $\mathcal{R}^{\tilde{P}^\star}(\text{cm}, \epsilon)$



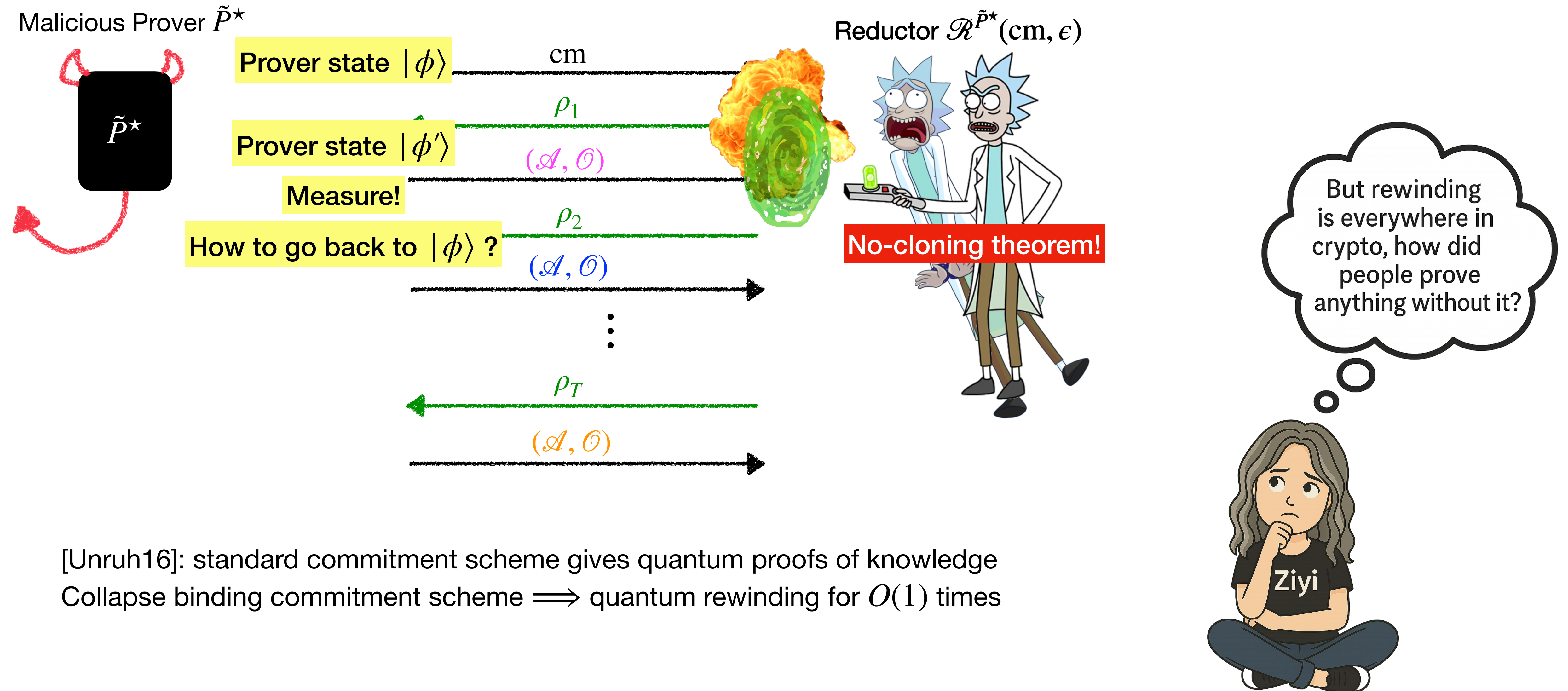
Black-box simulation of $\langle \tilde{P}^\star, V \rangle$



Uniformly sampled

Send measured outcome to V

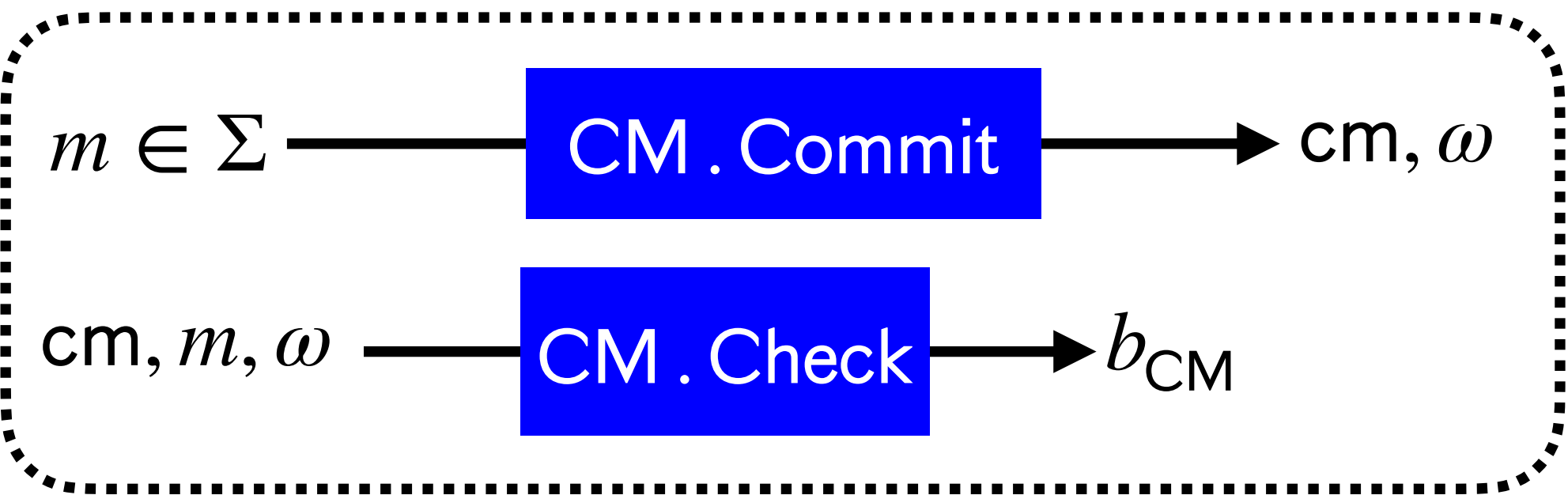
On quantum rewinding



[Unruh16]: standard commitment scheme gives quantum proofs of knowledge
 Collapse binding commitment scheme $\implies$ quantum rewinding for $O(1)$ times

Quantum rewinding with commitment schemes

Commitment scheme (CM)



Collapse binding

Undetectable measurement

$cm, (\mathcal{M}, \mathcal{W}) \leftarrow \text{Adv}$

Exp_0 : does nothing

Exp_1 : measure $\mathcal{M}$

$(\mathcal{M}, \mathcal{W}) \rightarrow \text{Adv}$

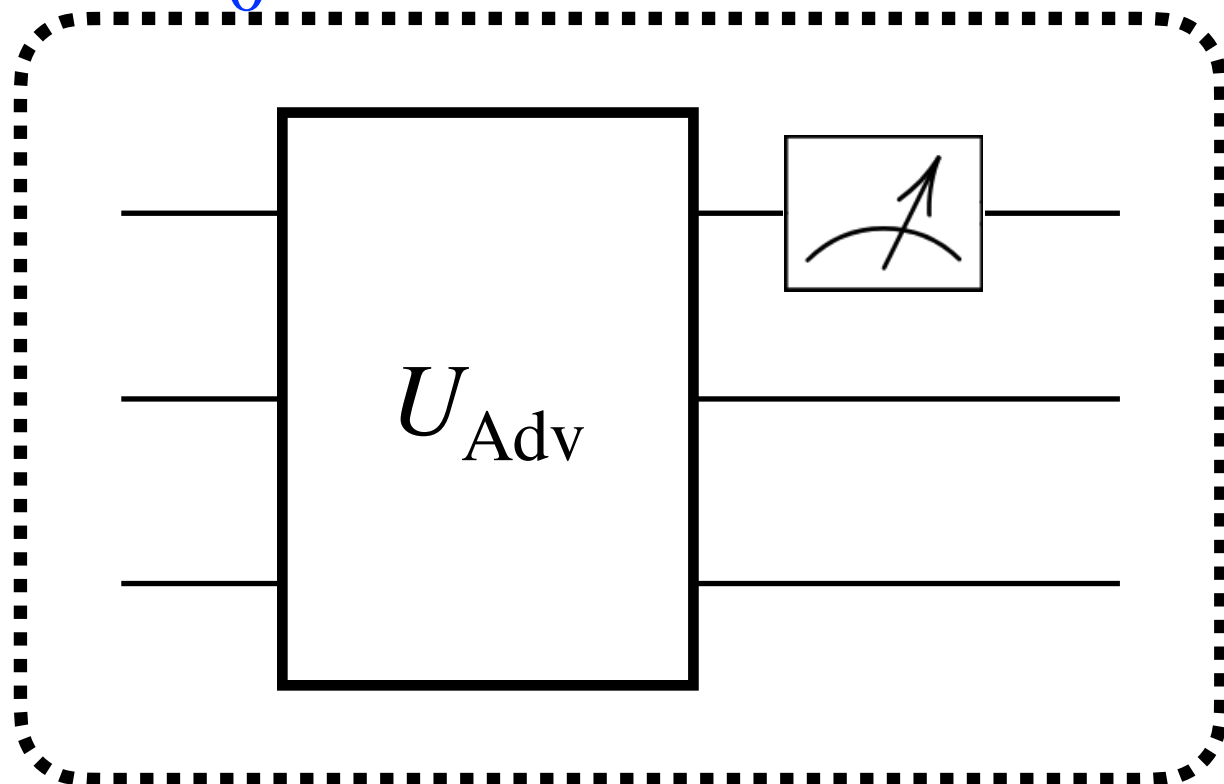
$\Pr[\text{Adv distinguishes } \text{Exp}_0 \text{ and } \text{Exp}_1] \leq \epsilon_{\text{CMCollapse}}^*$

Commitment register $\mathcal{C}$

Message register $\mathcal{M}$

Opening register $\mathcal{W}$

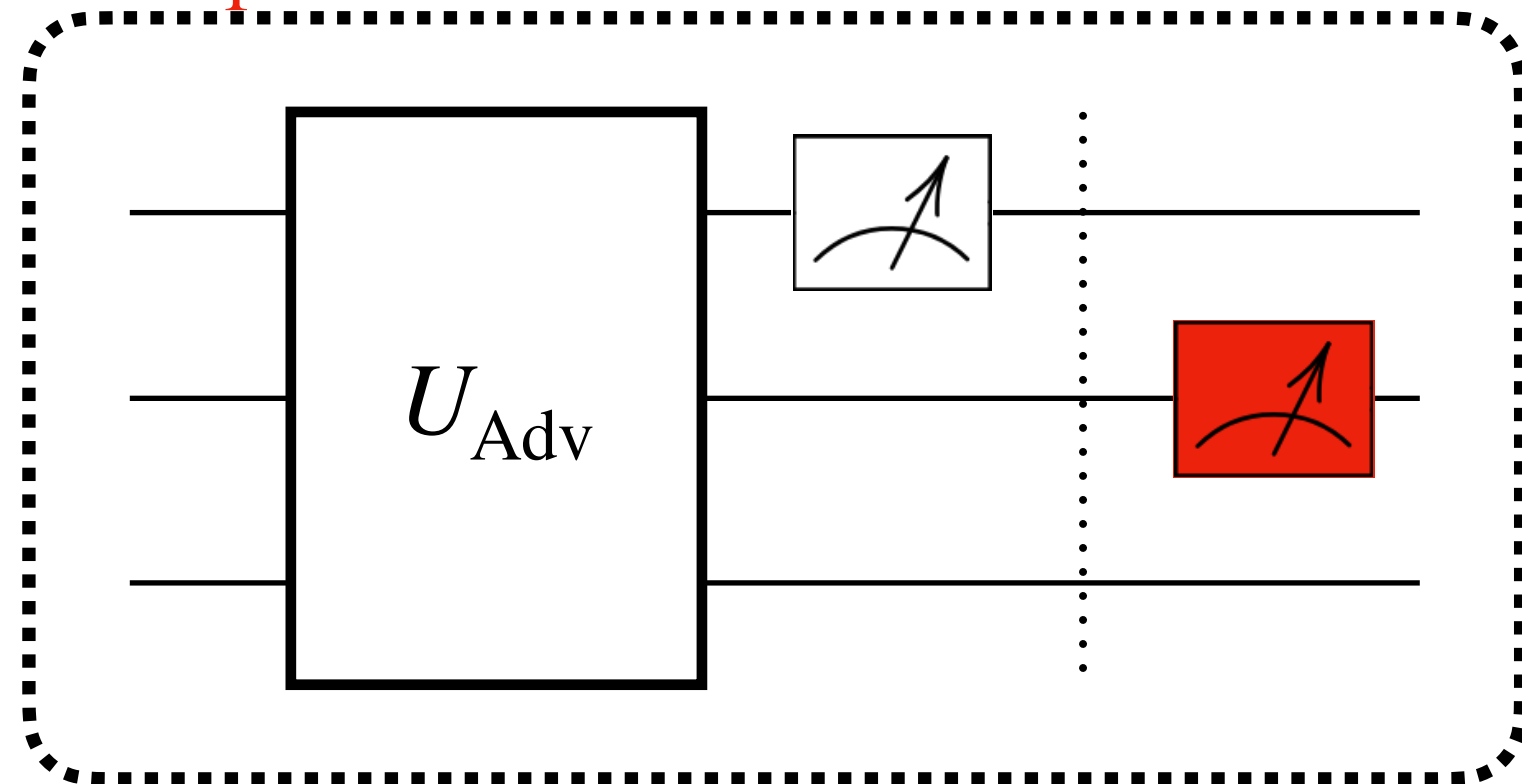
Exp_0



Computationally Indistinguishable

$\approx$

Exp_1



Collapse binding $\implies$ binding
(Adv cannot give different openings for one cm)

How about vector commitments?

CMSZ collapsing

$(\text{cm}, Q), (\mathcal{A}, \mathcal{O}) \leftarrow \text{Adv}$

Exp_0 : does nothing

Exp_1 : measure $(\mathcal{A}, \mathcal{O})$

$(\mathcal{A}, \mathcal{O}) \rightarrow \text{Adv}$

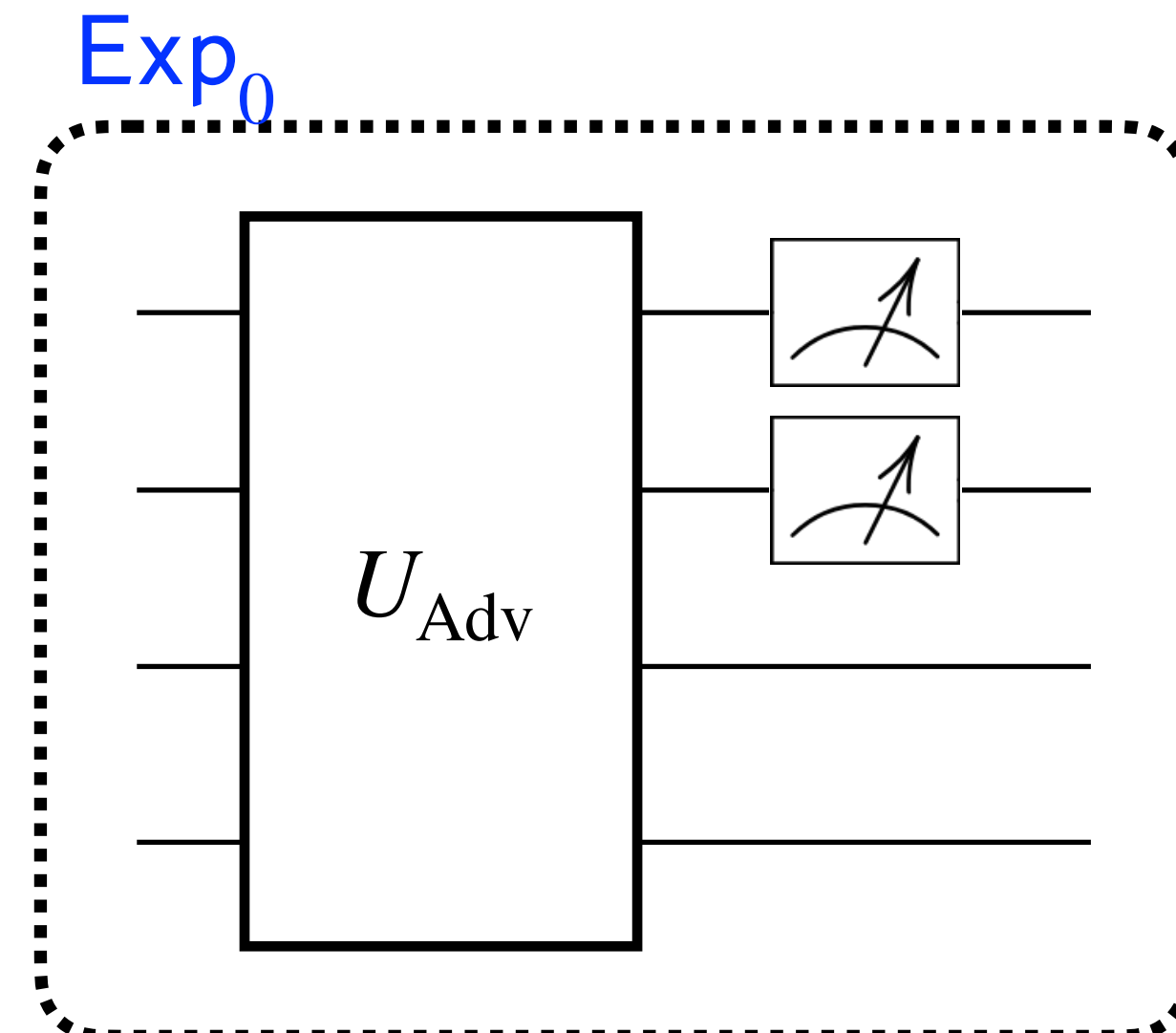
$\Pr[\text{Adv distinguishes } \text{Exp}_0 \text{ and } \text{Exp}_1] \leq \epsilon_{\text{CMSZCollapse}}^*$

Why measure $\mathcal{O}$? CM only measure $\mathcal{M}$
[CMSZ21] security analysis needs $\mathcal{O}$

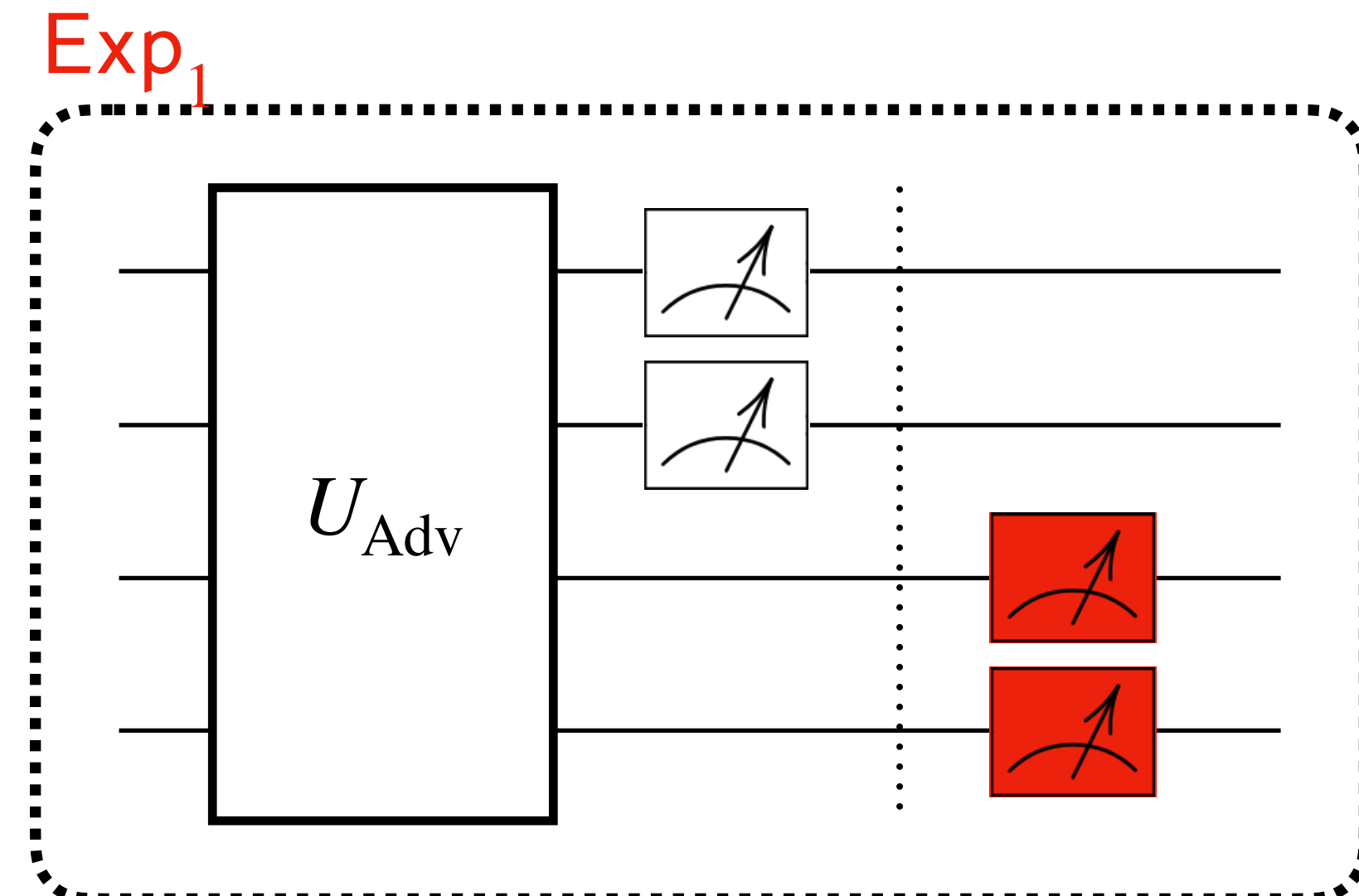
Issue: does not imply position binding

- CMSZ collapsing - one single query set
- Position binding - two query sets Q, Q'

Commitment register $\mathcal{C}$
Query register Q
Answer register $\mathcal{A}$
Opening register $\mathcal{O}$



Computationally Indistinguishable
 $\approx$



Post-quantum security of Kilian's protocol

Queries only depend on randomness

Theorem [CMSZ21]. $\forall$ non-adaptive PCP, VC (negligible ϵ_{PQPB}^* , negligible $\epsilon_{\text{CMSZCollapse}}^*$),
$$\epsilon_{\text{ARG}}^* \leq \epsilon_{\text{PCP}} + \text{negl}$$

Can we get a more robust VC collapsing def?
(VC collapsing that implies position binding)

PCP is not concretely efficient - Can we use IOPs?

Can we get concrete bound as classical case?

Can we handle adaptive PCPs?

**A new collapsing definition for VC:
Collapse position binding**

Naive attempt: openings to different subsets

Naive-attempt collapsing

$\text{cm}, (\mathcal{Q}, \mathcal{A}, \mathcal{O}) \leftarrow \text{Adv}$

Impossible to achieve!

Exp_0 : does nothing

Exp_1 : measure $(\mathcal{Q}, \mathcal{A}, \mathcal{O})$

$(\mathcal{Q}, \mathcal{A}, \mathcal{O}) \rightarrow \text{Adv}$

$\Pr[\text{Adv distinguishes } \text{Exp}_0 \text{ and } \text{Exp}_1] \leq \epsilon_{\text{NaiveCollapse}}^*$

Recall:

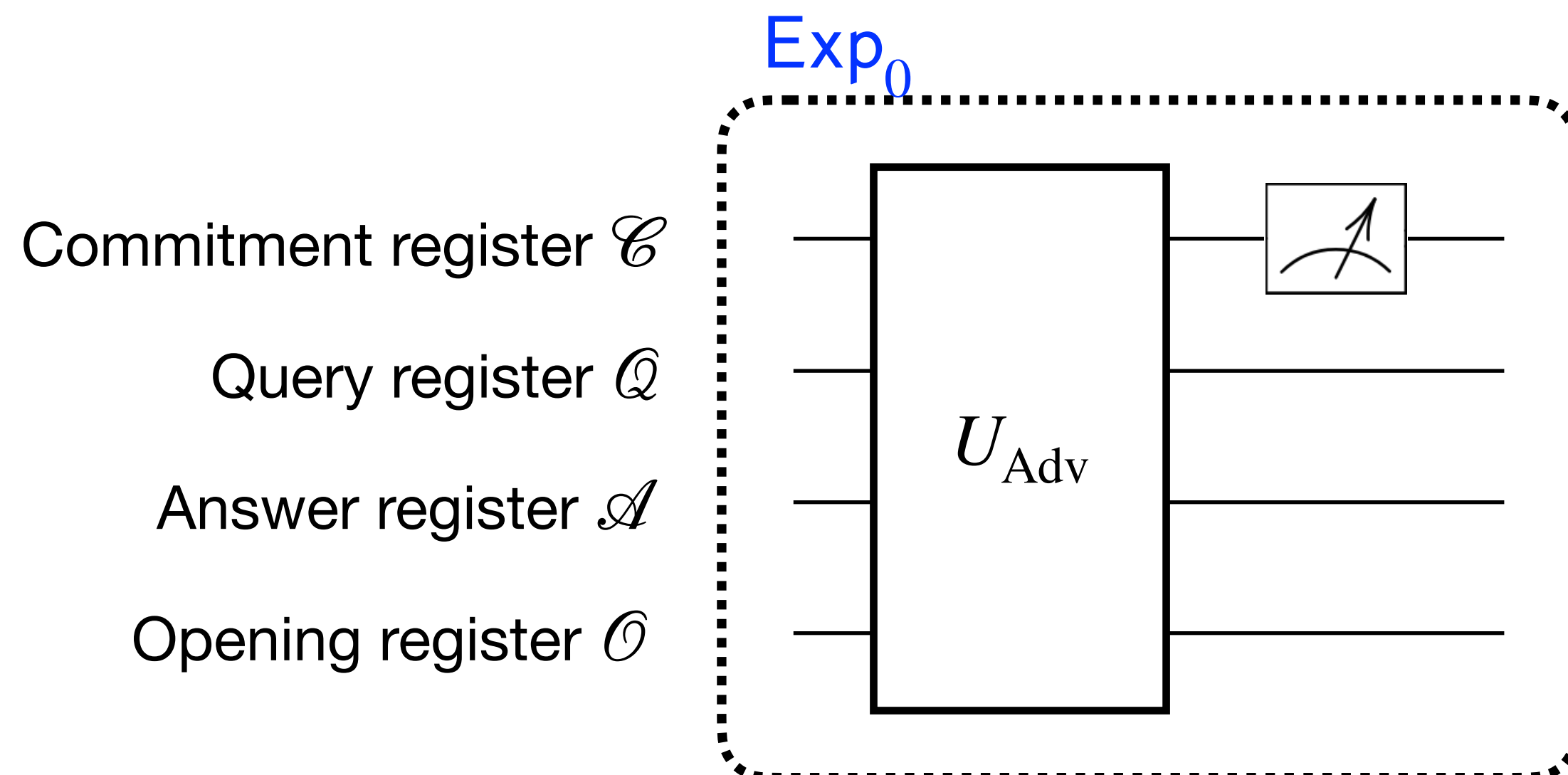
- CMSZ collapsing - one single query set
- Position binding - two query sets $\mathcal{Q}, \mathcal{Q}'$

Assume cm has two valid openings $(\mathcal{Q}, \text{ans}, \text{pf}), (\mathcal{Q}', \text{ans}', \text{pf}')$

$\text{Adv} \rightarrow \text{cm}, | \mathcal{Q}, \text{ans}, \text{pf} \rangle + | \mathcal{Q}', \text{ans}', \text{pf}' \rangle$

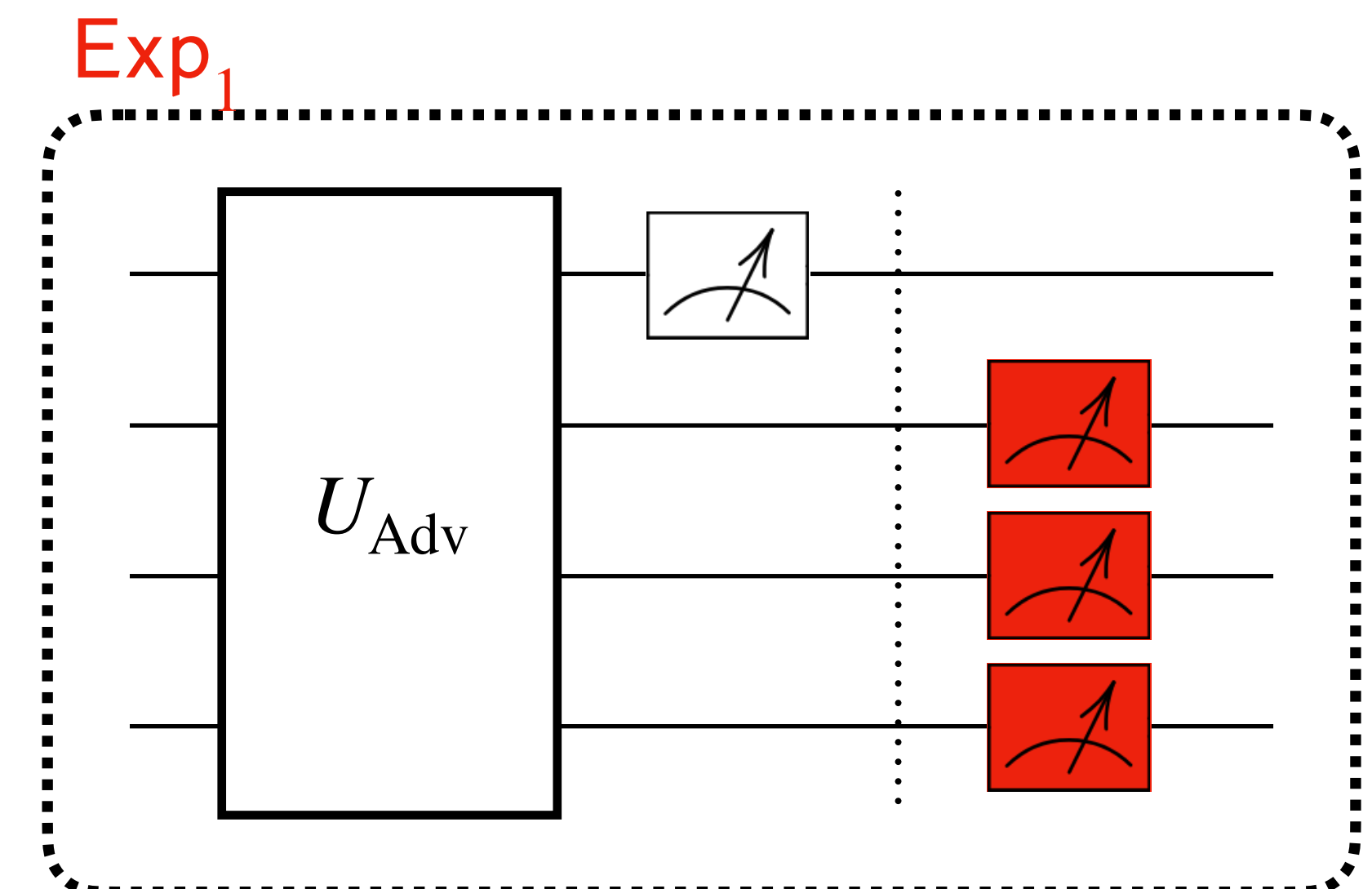
Measuring $(\mathcal{Q}, \mathcal{A}, \mathcal{O}) \Rightarrow (\mathcal{Q}, \text{ans}, \text{pf}) \text{ or } (\mathcal{Q}', \text{ans}', \text{pf}')$

$\Rightarrow$ Easily distinguishable from uniform superposition



Computationally Indistinguishable

$\approx$



Collapse position binding

Lifting from commitment schemes

Collapse position binding

$(\text{cm}, \text{idx}), (\mathcal{A}, \mathcal{O}) \leftarrow \text{Adv}$

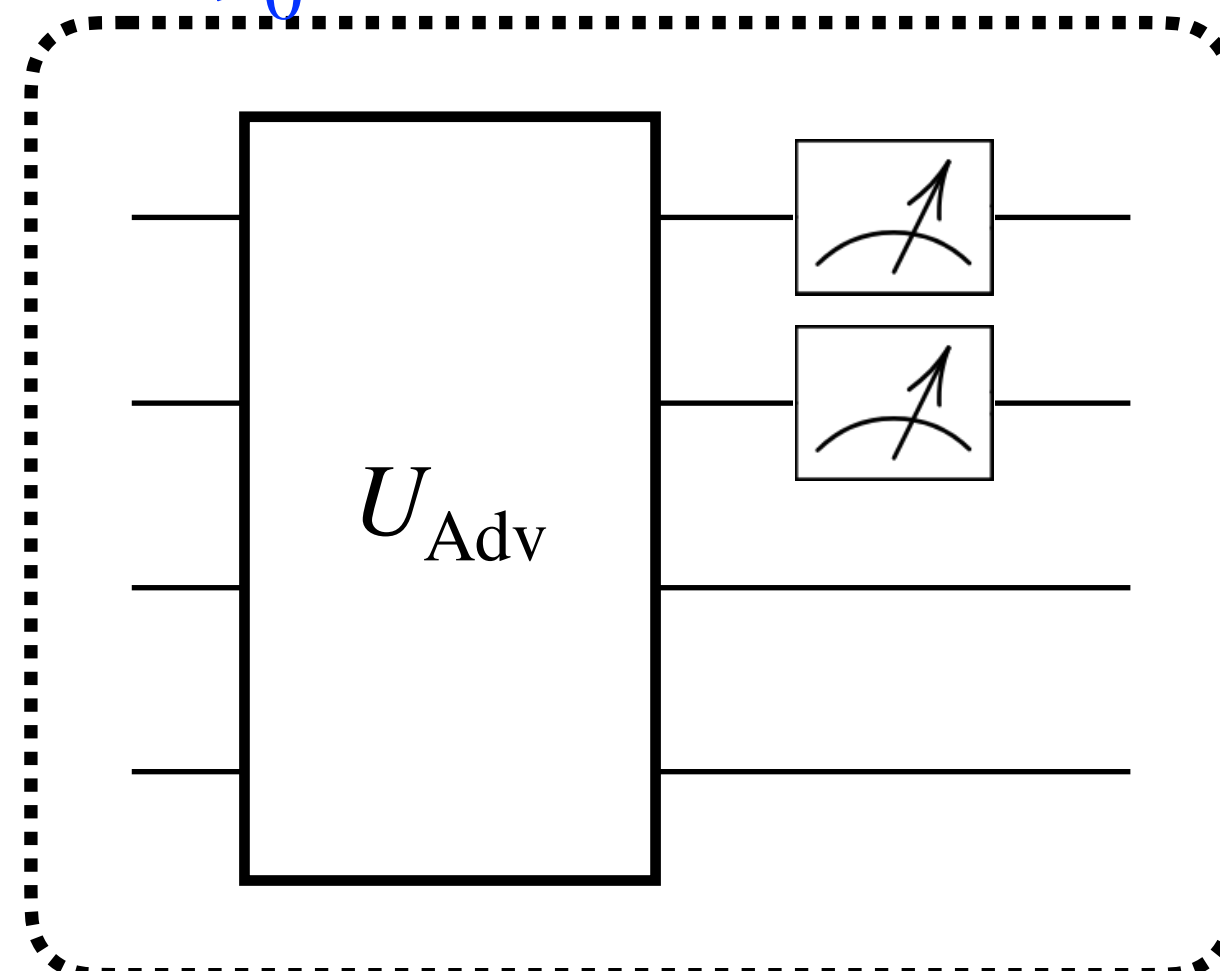
Exp_0 : does nothing

Exp_1 : measure $\mathcal{A}$ at location idx

$(\mathcal{A}, \mathcal{O}) \rightarrow \text{Adv}$

$\Pr[\text{Adv distinguishes } \text{Exp}_0 \text{ and } \text{Exp}_1] \leq \epsilon_{\text{VCCollapsePB}}^*$

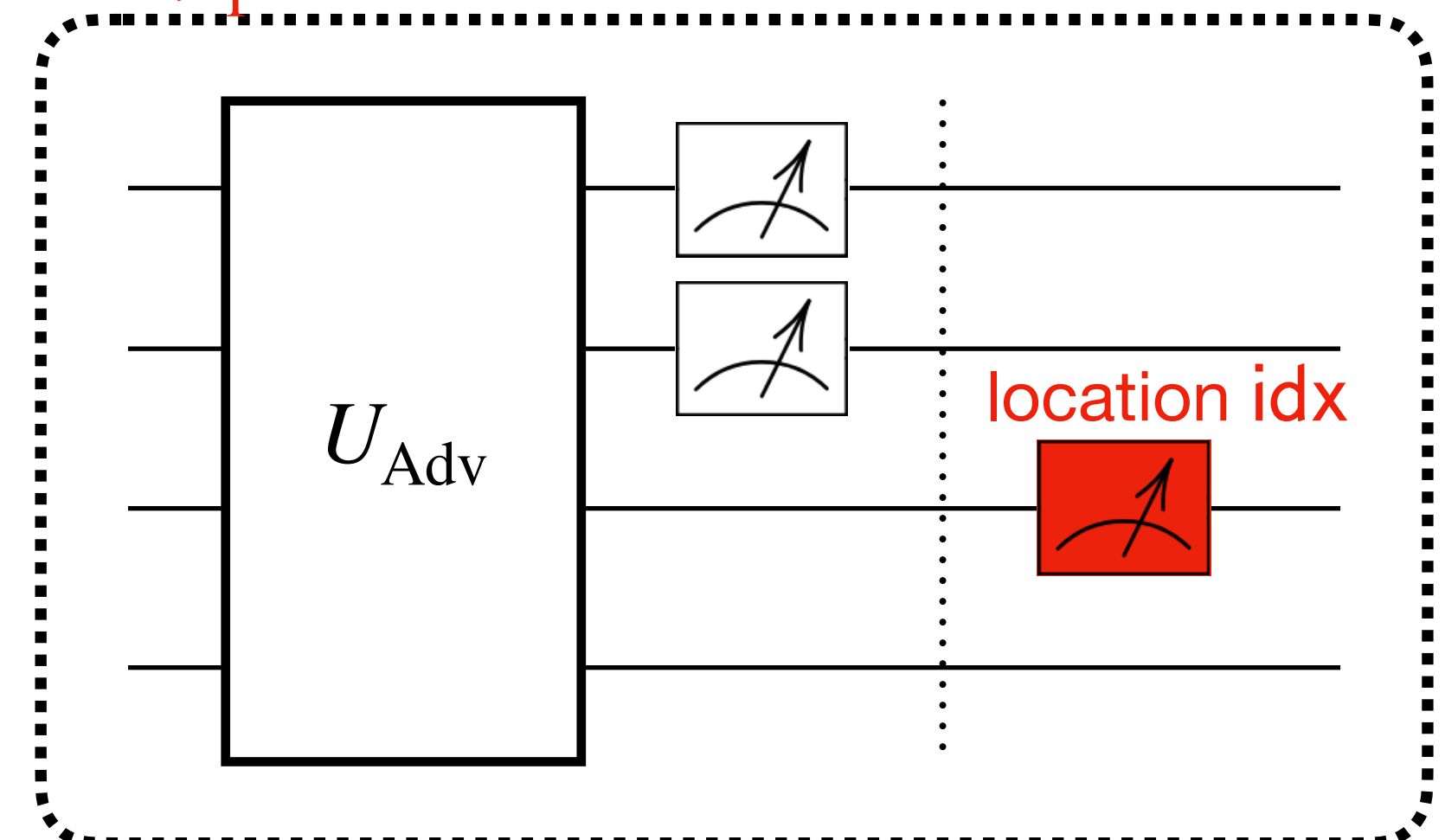
Exp_0



Computationally Indistinguishable

$\approx$

Exp_1



Vector commitment scheme VC for length ℓ

$\forall i \in [\ell], \text{commitment scheme } \text{CM}_i$

Known:

- VC position binding $\iff \forall i, \text{CM}_i$ binding
- CM collapse binding $\implies$ CM binding

Goal: VC collapse position binding $\iff \forall i, \text{CM}_i$ collapse binding

VC collapse position binding $\implies$ VC position binding

Improved post-quantum security of Kilian's protocol

Queries only depend on randomness

Theorem [CMSZ21]. $\forall$ non-adaptive PCP, VC (negligible ϵ_{PCPB}^* , negligible $\epsilon_{\text{VCCollapsePB}}^*$)

$$\epsilon_{\text{ARG}}^* \leq \epsilon_{\text{PCP}} + \text{negl}$$

Can we get a more robust VC collapsing def?
(VC collapsing that implies position binding)

YES!

PCP is not concretely efficient - Can we use IOPs?

Can we get concrete bound as classical case?

Can we handle adaptive PCPs?

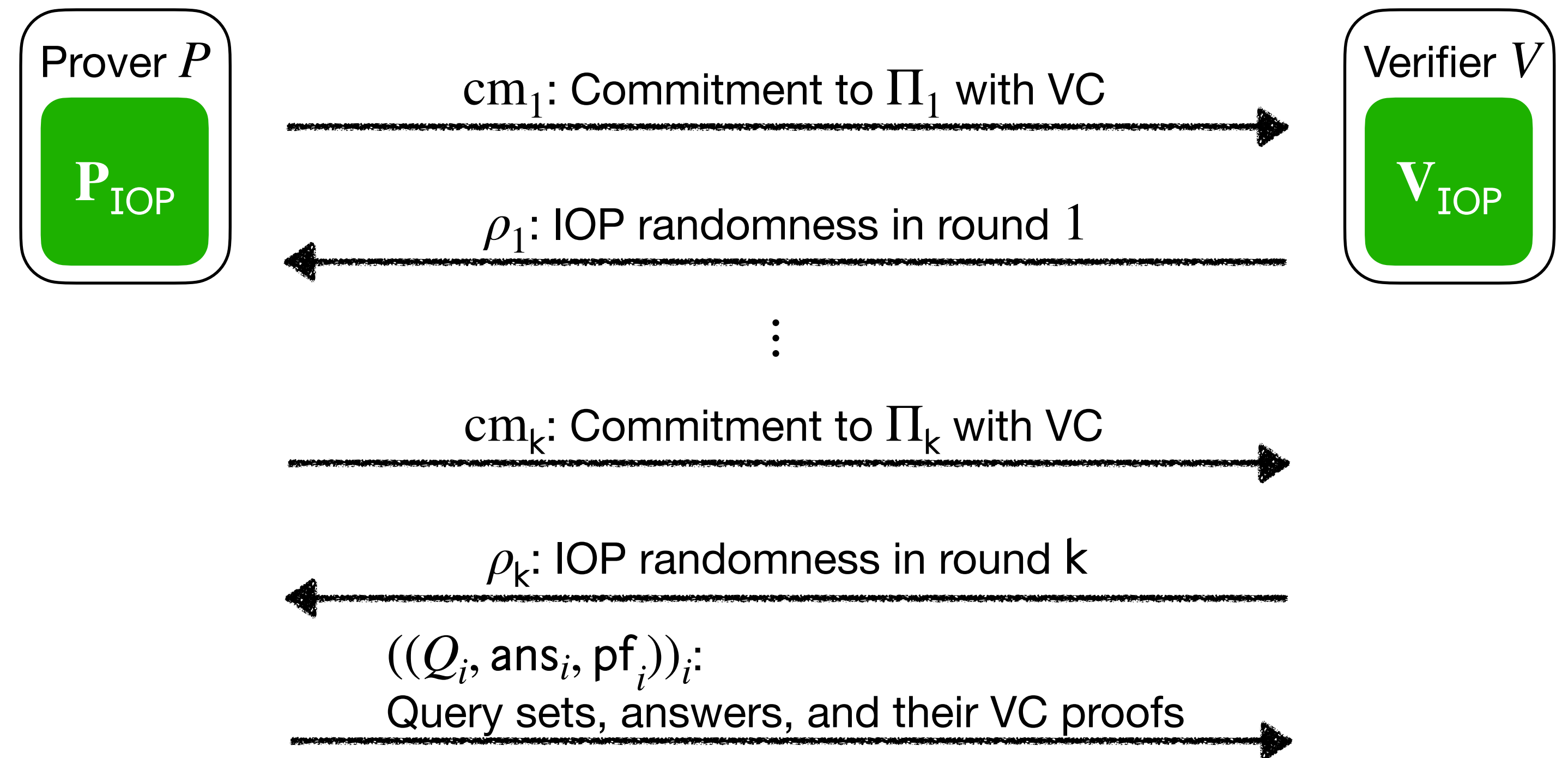
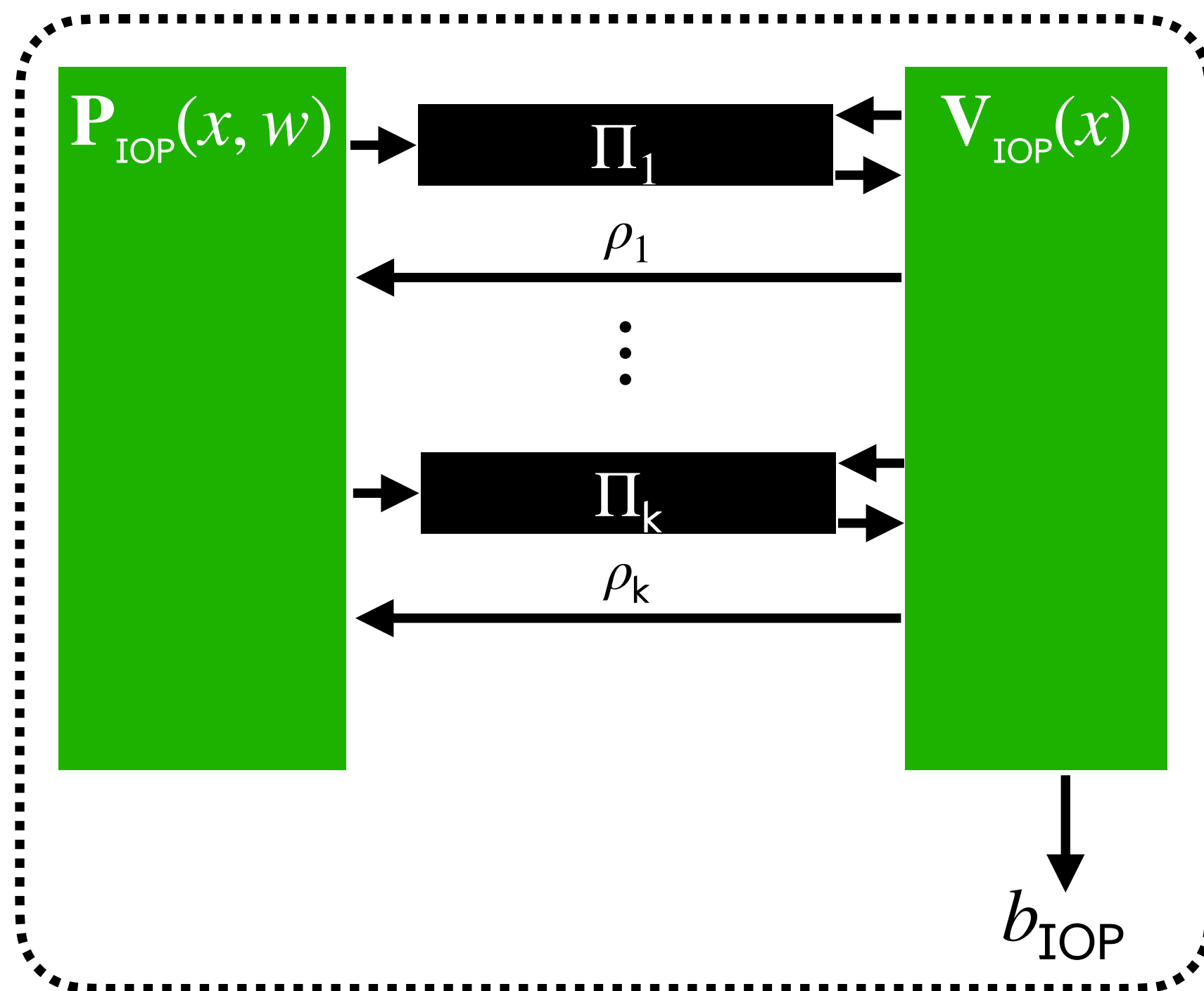
**IBCS protocol:
Using IOPs instead of PCPs**

IBCS protocol

Existing PCPs are not concretely efficient: prover time too big

People use IOPs

Public-coin **interactive oracle proof** (IOP)



Our result

Queries depend on randomness and answers to queries to previous proofs

Theorem. $\forall$ semi-adaptive IOP, VC, $\epsilon > 0$,

$$\epsilon_{\text{ARG}}^{\star}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + k \cdot \ell_{\max} \cdot q_{\max} \cdot \epsilon_{\text{VCCollapsePB}}^{\star}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = \text{poly}(\ell/\epsilon) \cdot t_{\text{ARG}}.$$

Extra $\ell_{\max} \cdot q_{\max}$ factor: cost of quantum rewinding

Quantum rewinding can fail
 $\text{poly}(\ell/\epsilon)$ attempts $\implies \ell/\epsilon$ valid rewindings

IBCS soundness [CDGS23,CGKY25]: $\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + k \cdot \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon$, where $t_{\text{VC}} = O(t_{\text{ARG}} \cdot \ell/\epsilon)$.

Corollary: post-quantum secure succinct arguments in the standard model (no oracles),
 with **the best asymptotic complexity known**.

Corollary for Kilian's protocol. $\forall$ adaptive PCP, VC, $\epsilon > 0$,

$$\epsilon_{\text{ARG}}^{\star}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + \ell \cdot q \cdot \epsilon_{\text{VCCollapsePB}}^{\star}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = \text{poly}(\ell/\epsilon) \cdot t_{\text{ARG}}.$$

Theorem [CMSZ21]. $\forall$ non-adaptive PCP, VC (negligible $\epsilon_{\text{PQPB}}^{\star}$, negligible $\epsilon_{\text{CMSZCollapse}}^{\star}$),

$$\epsilon_{\text{ARG}}^{\star} \leq \epsilon_{\text{PCP}} + \text{negl}$$

YES!

Can we get a more robust VC collapsing def?

PCP is not concretely efficient - Can we use IOPs?

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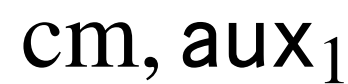
Post-quantum security for Kilian

Starting point: [CMSZ21]

[CMSZ21] reductor



Reductor $\mathcal{R}^{\tilde{P}}(\text{cm}, \epsilon)$

 ρ_1 $(\mathcal{A}, \mathcal{O}, \text{aux}'_1)$

Measure if $V(\rho_1, \mathcal{A}, \mathcal{C}) = 1$

```
aux2 ← RepairState(aux'1)
```

 ρ_2
$$(\mathcal{A}, \mathcal{O}, \mathbf{aux}'_2)$$

Measure if $V(\rho_2, \mathcal{A}, \mathcal{C}) = 1$

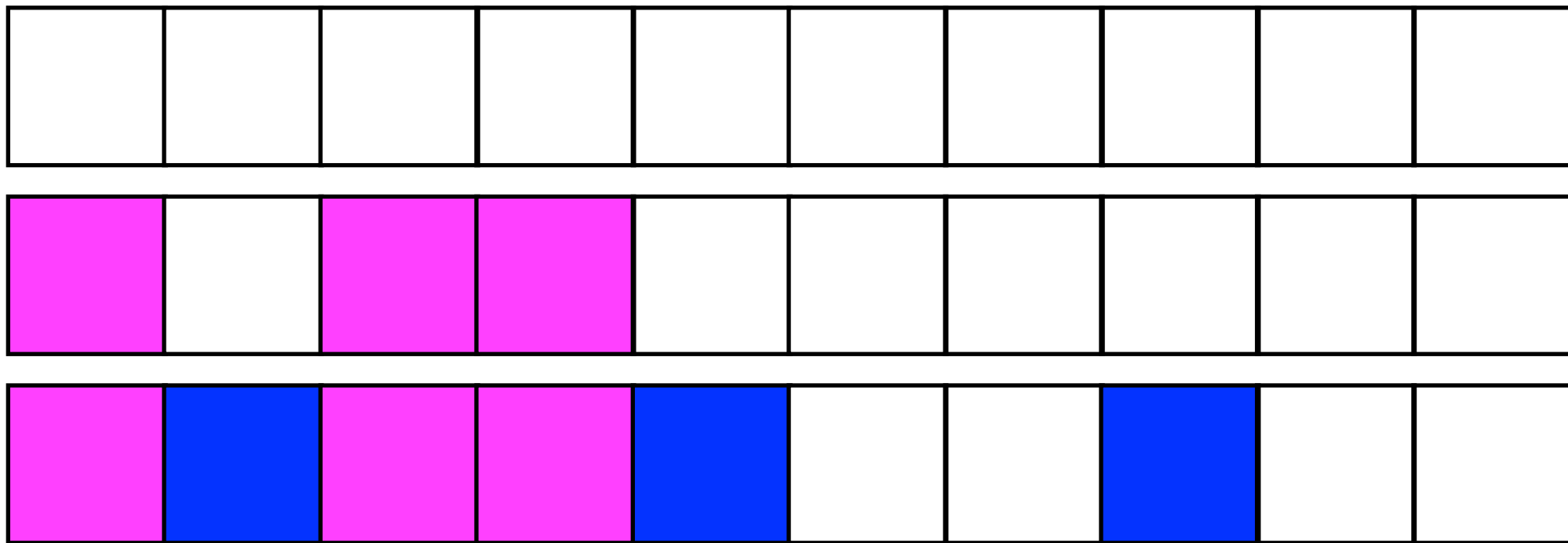
```
aux3 ← RepairState(aux'2)
```

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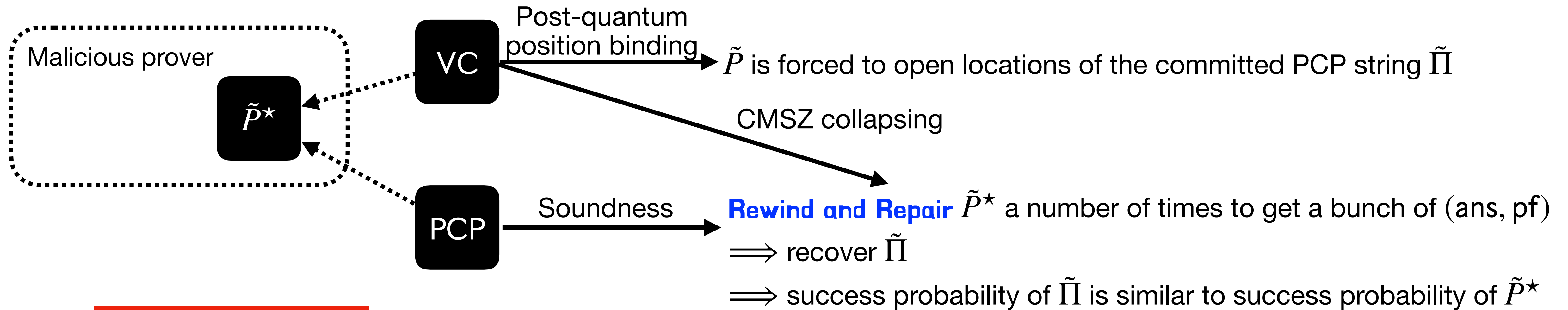
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RepairState preserves $\text{SuccProb}(\tilde{P}^\star)$

Recover $\tilde{\Pi}$ 

[CMSZ21] security reduction

Goal: relate the soundness error of Kilian[PCP, VC]
to the soundness error of PCP and the post-quantum security of VC.



Doesn't work for IOP!

How many possible IOP strings?

- Alphabet Σ
 - Proof length ℓ
 - Verifier randomness complexity r
- $\Rightarrow |\{f: \{0,1\}^r \rightarrow \Sigma^\ell\}| = |\Sigma|^{\ell \cdot 2^r}$

RepairState preserves SuccProb($\tilde{P}^*$)

Consider $\hat{\Pi}$ s.t. $\text{SuccProb}(\hat{\Pi}) < \text{SuccProb}(\tilde{P}^*)/20$

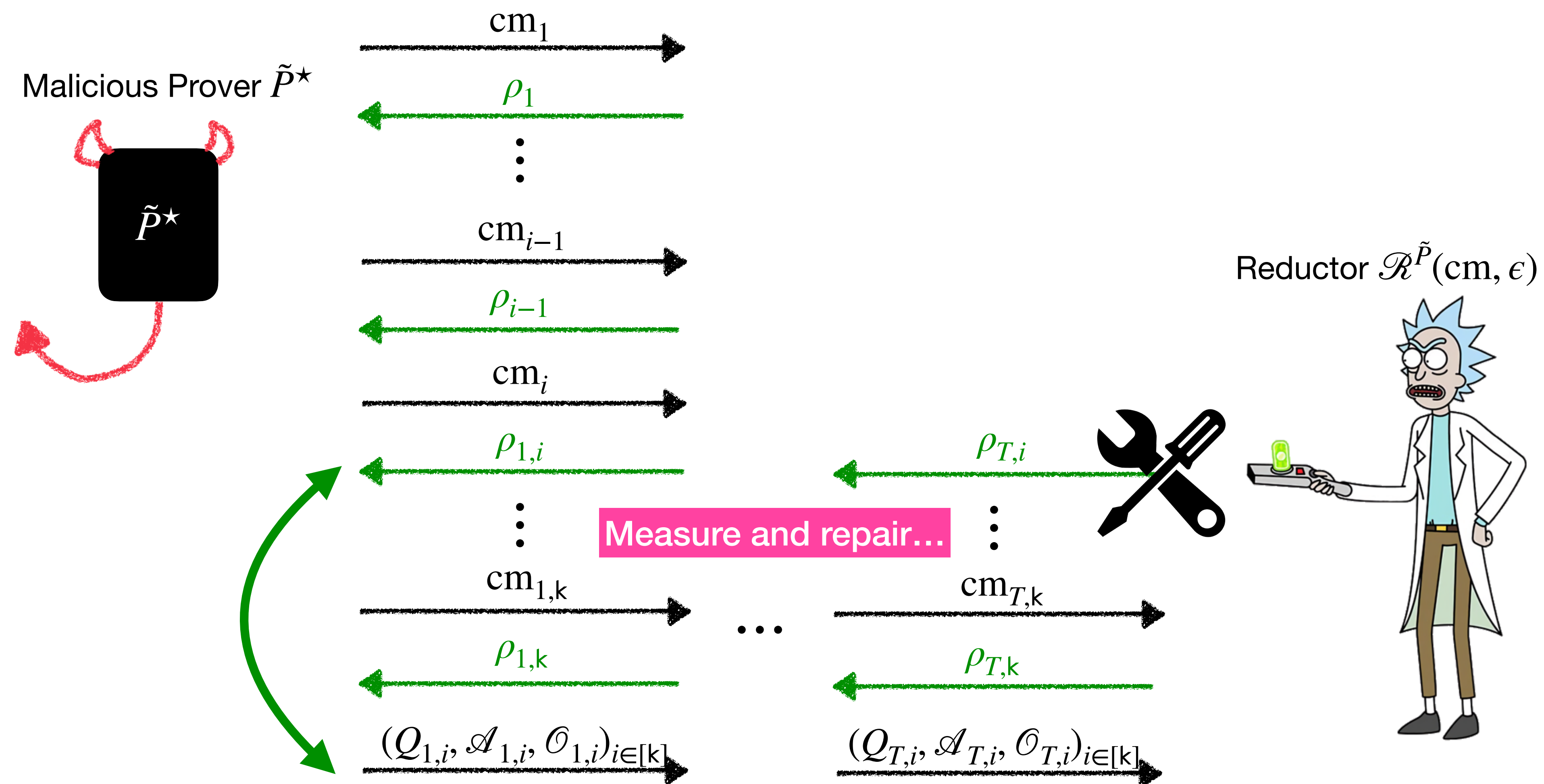
$\Rightarrow$ (Chernoff) $\Pr[\tilde{\Pi} = \hat{\Pi}] \ll |\Sigma|^{-\ell}$

$\Rightarrow$ (Union bound) $\Pr[\text{SuccProb}(\tilde{\Pi}) < \text{SuccProb}(\tilde{P}^*)/20]$ very small

$\Rightarrow \epsilon_{\text{ARG}}^* = \text{SuccProb}(\tilde{P}^*) \leq \epsilon_{\text{PCP}} + \text{negl}$

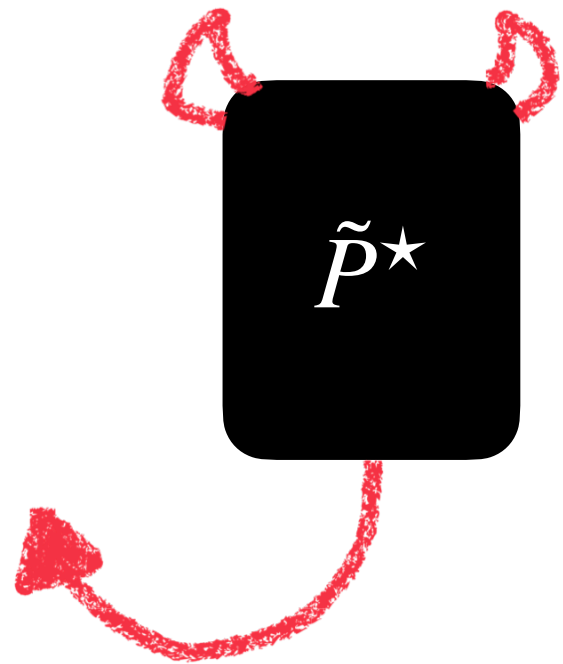
Our security reduction

Our quantum reductor



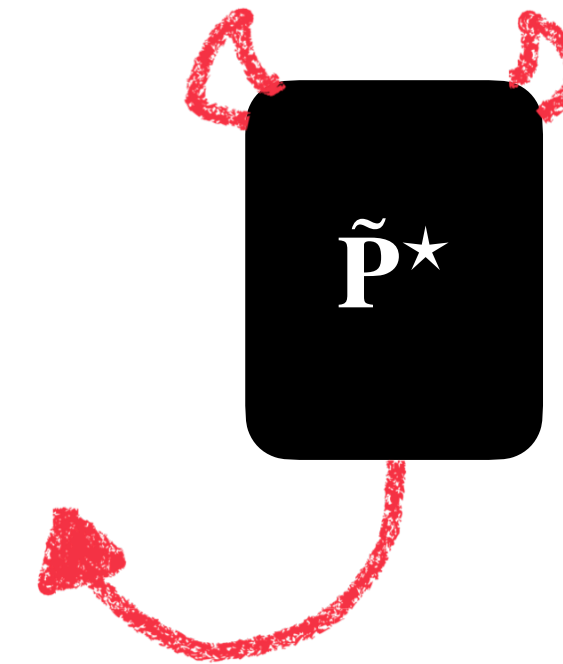
Hybrid argument

Malicious Prover $\tilde{P}^\star$



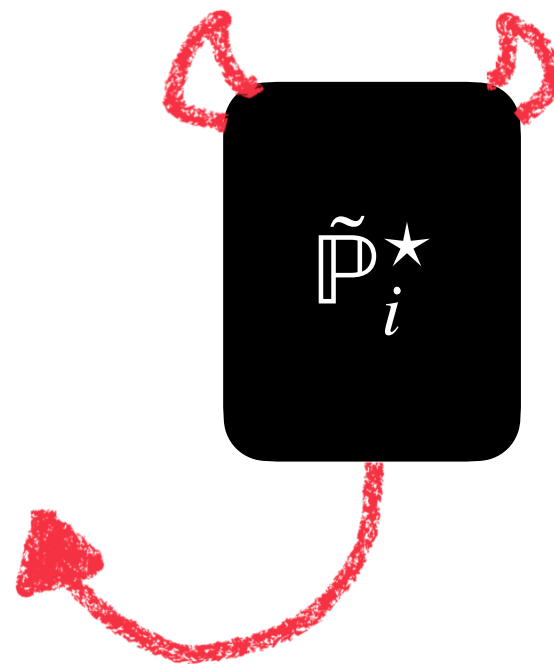
Security reduction using $\mathcal{R}^{\tilde{P}^\star}$

Malicious IOP Prover $\tilde{P}^\star$



Goal: $\text{SuccProb}(\tilde{P}^\star) \approx \text{SuccProb}(\tilde{P}^\star)$

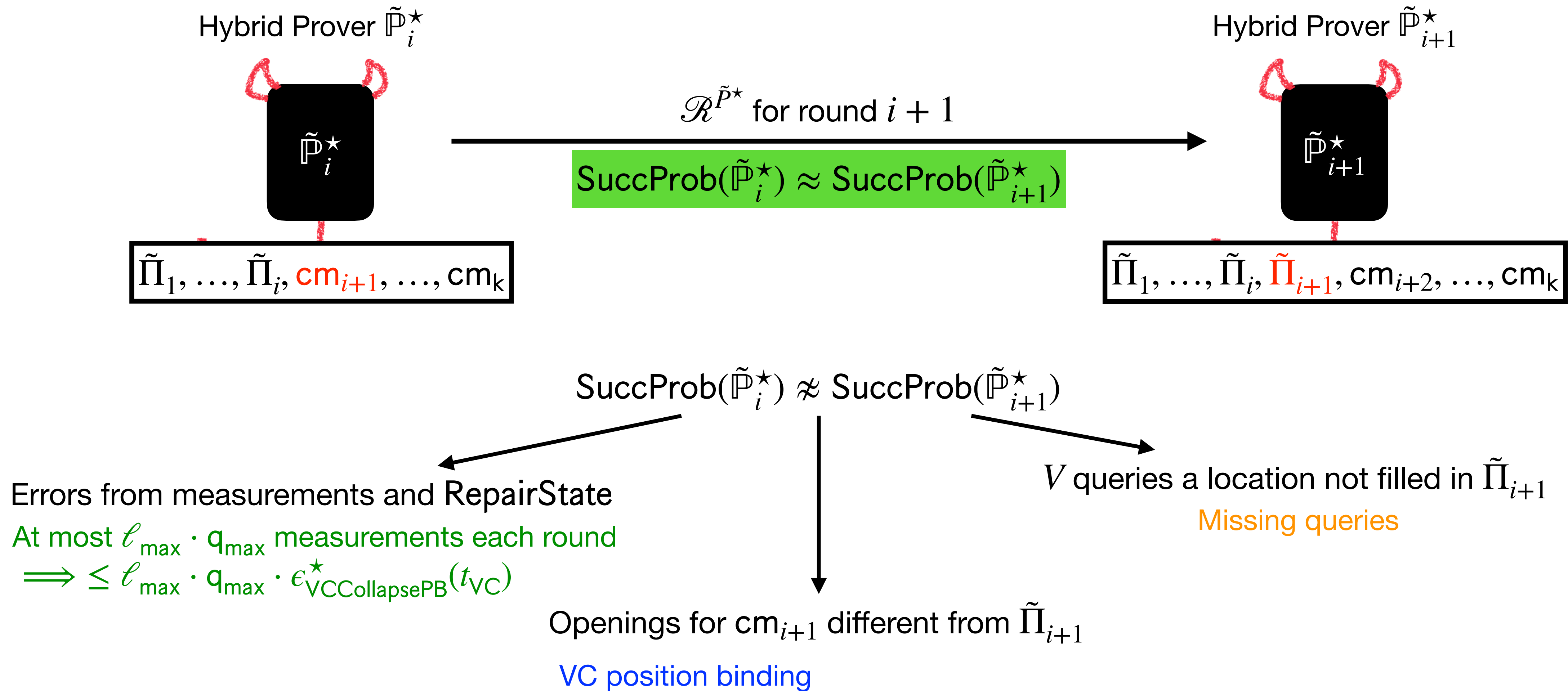
Hybrid Prover $\tilde{P}_i^\star$



- First i rounds: output IOP strings output by $\mathcal{R}^{\tilde{P}^\star}$
- Rest of the rounds: same as $\tilde{P}^\star$

We show: $\forall i, \text{SuccProb}(\tilde{P}_i^\star) \approx \text{SuccProb}(\tilde{P}_{i+1}^\star)$

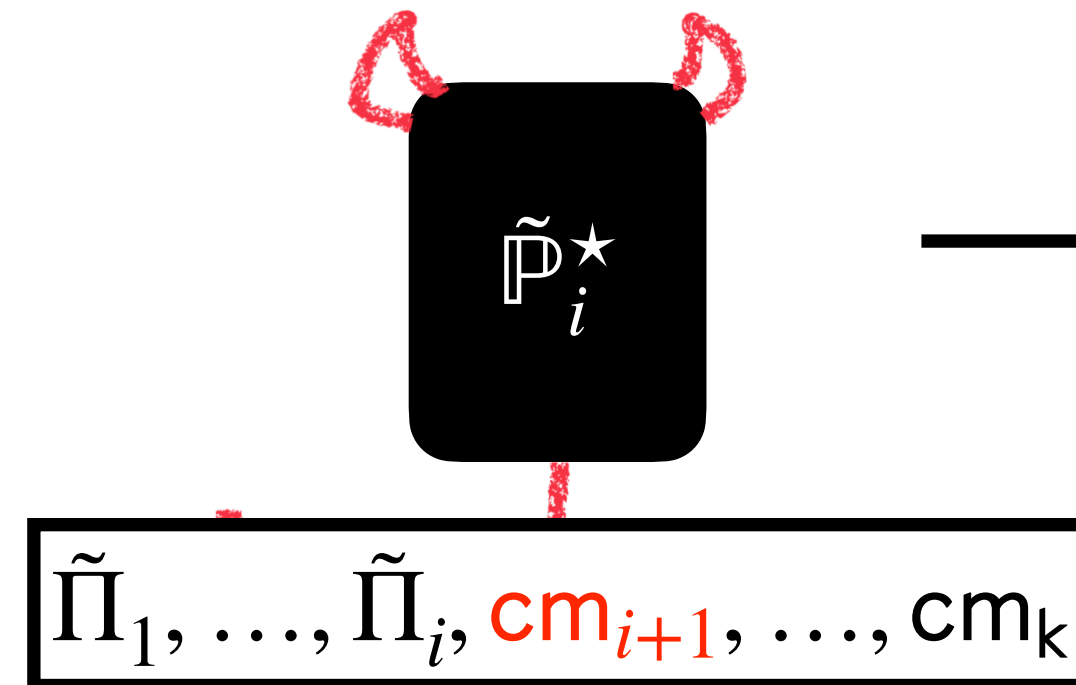
Our security reduction



Missing queries

V queries a location not filled in $\tilde{\Pi}_{i+1}$

Hybrid Prover $\tilde{P}_i^*$

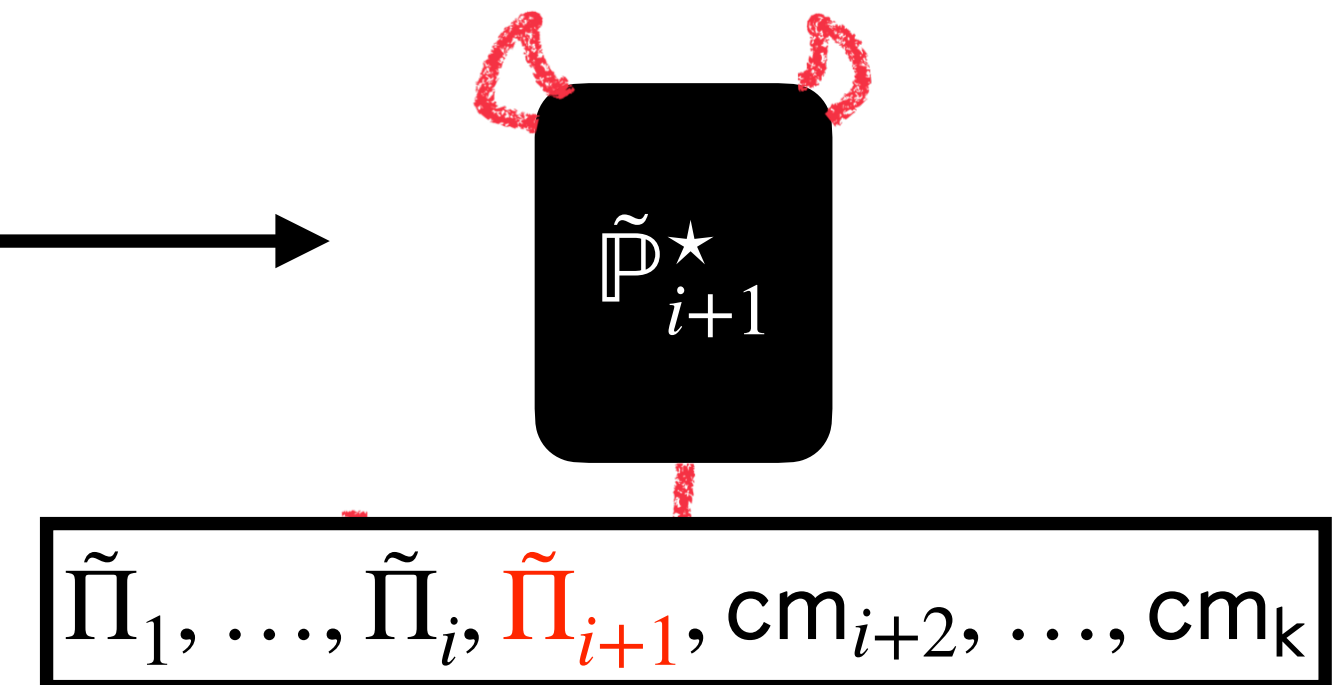


Run $\tilde{P}_i^*$ one more time, get openings for cm_{i+1}
 $\implies$ The $(T + 1)$ -th rewind

$\mathcal{R}^{\tilde{P}^*}$ for round $i + 1$

$$\text{SuccProb}(\tilde{P}_i^*) \approx \text{SuccProb}(\tilde{P}_{i+1}^*)$$

Hybrid Prover $\tilde{P}_{i+1}^*$



Rewind $\tilde{P}_i^*$ T times to get $\tilde{\Pi}_{i+1}$

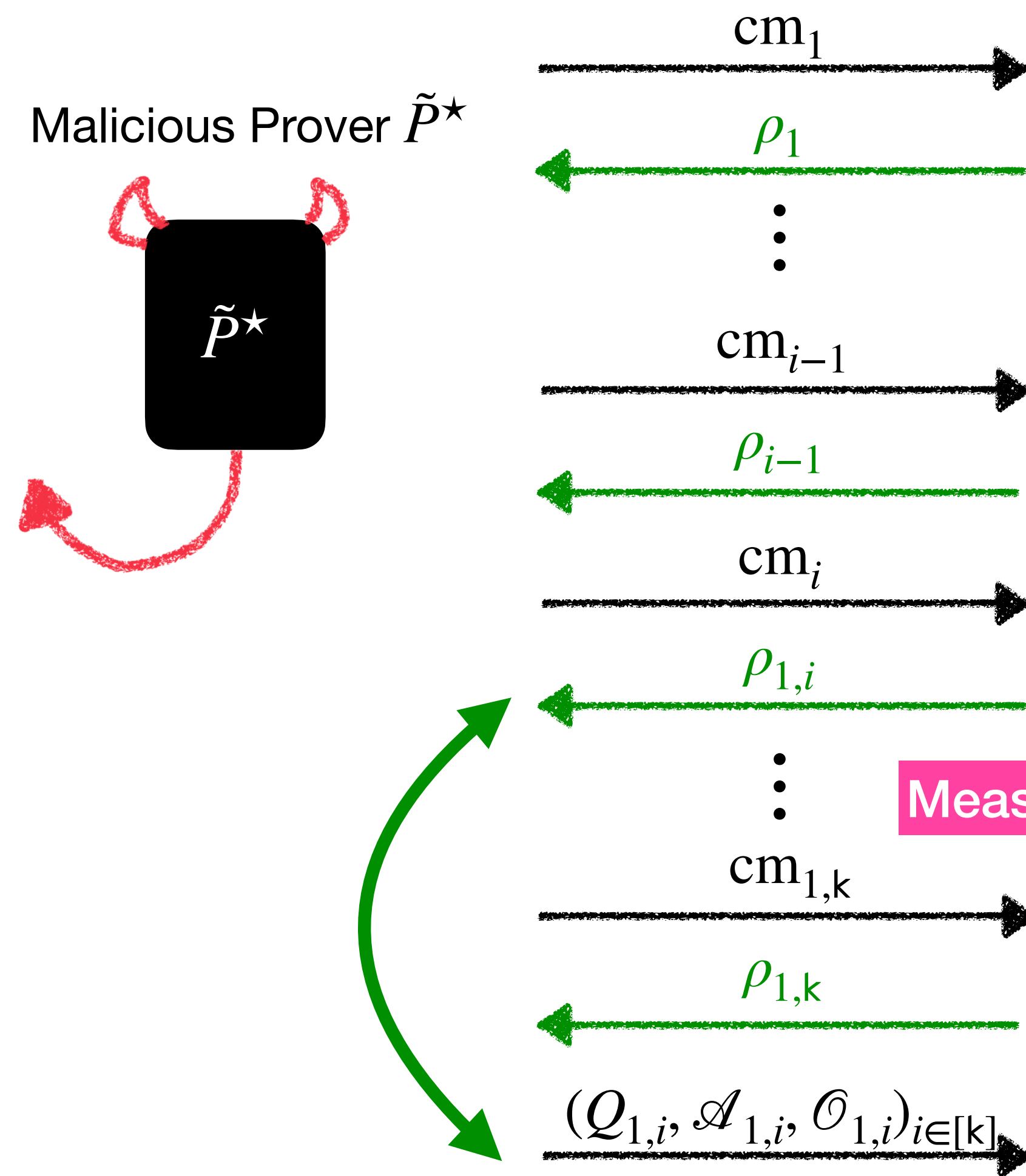
Classical approach

- δ_q : prob $q \in [\ell_i]$ queried by V and correctly opened by $\tilde{P}^*$
- $\Pr[\exists q, \tilde{\Pi}_{i+1}[q] \text{ unfilled}, q \text{ queried with valid opening}] \leq \ell_i \cdot \delta_q (1 - \delta_q)^T \leq \ell_i / T$
- Setting T to get desired bound

Doesn't work for quantum!

**RepairState only preserves $\text{SuccProb}(\tilde{P}^*)$
 $\implies$ does not preserve δ_q**

Random stopping time



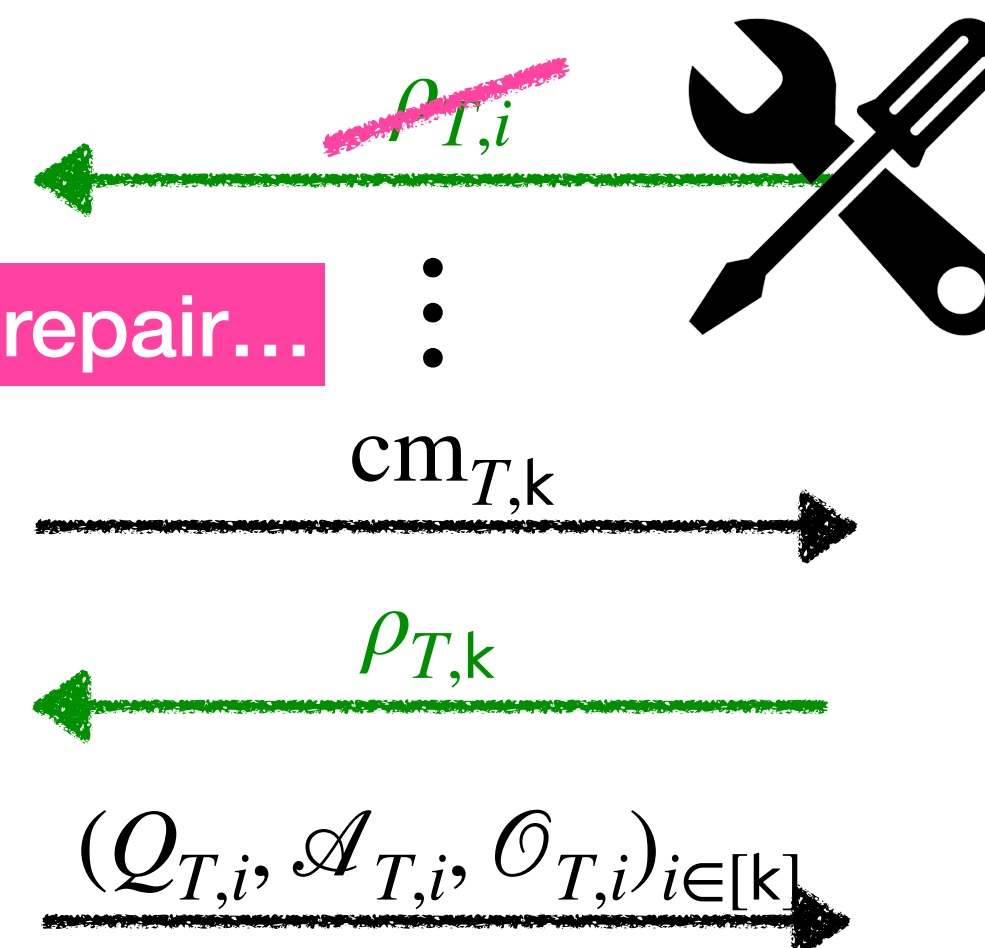
Key: total number of locations $q \in [\ell_i]$ filled in by $\mathcal{R}$ is ℓ_i

$$\zeta_j: \mathbb{I}[Q_{j+1,i} \neq \cup_{c \leq j} Q_{c,i}]$$

$$\implies \Pr[\text{missing queries}] = \mathbb{E}[\zeta_t]$$

$$(\text{Observation}) \mathbb{E}[\zeta_t] = 1/T \cdot \sum_{j \leq T} \mathbb{E}[\zeta_j] \leq \ell_i/T$$

Sample $t \leftarrow [T]$
Rewind for t times



Reductor $\mathcal{R}^{\tilde{P}^*}(\text{cm}, \epsilon)$



Why not adaptive IOP?

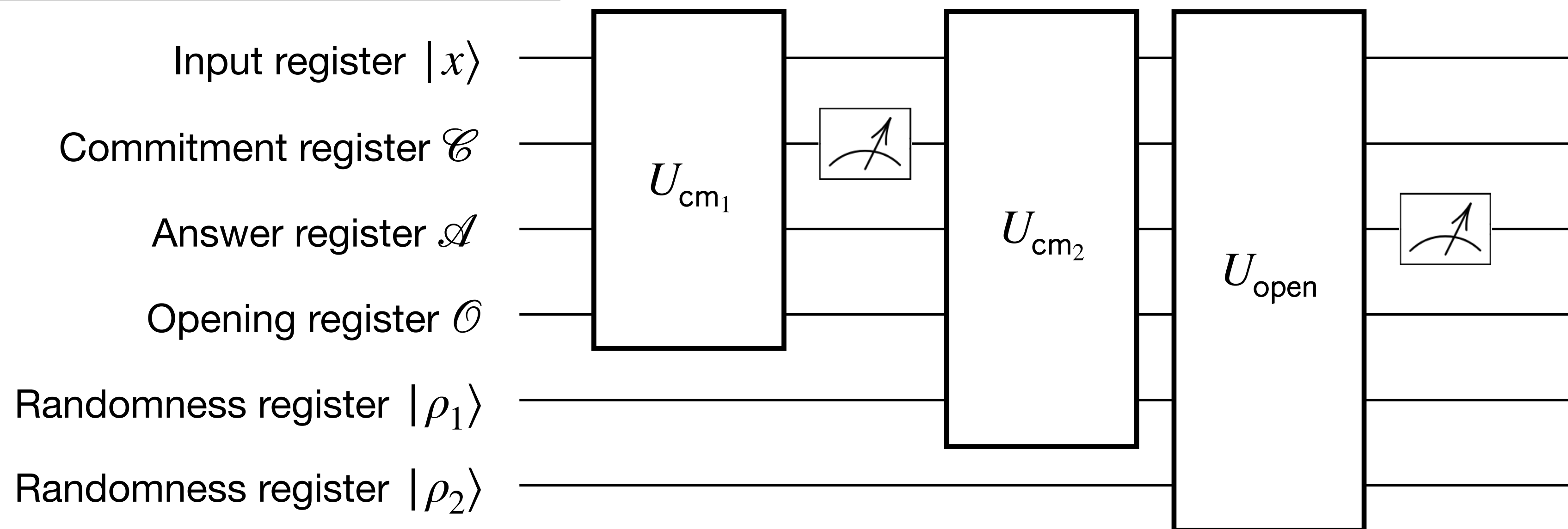
Queries to Π_1 depends on answers from Π_2

$(Q_1, \mathcal{A}_1, \mathcal{O}_1)$ and $(Q_2, \mathcal{A}_2, \mathcal{O}_2)$ entangled

$\implies$ Measuring $(Q_1, \mathcal{A}_1, \mathcal{O}_1)$ collapses $(Q_2, \mathcal{A}_2, \mathcal{O}_2)$

Collapse position binding **does not allow** measurement of $(Q_2, \mathcal{A}_2, \mathcal{O}_2)$

When $\mathcal{R}^{\tilde{P}^*}$ rewinds to get $\tilde{\Pi}_1 \dots$



Collapse position binding

$(cm, idx), (\mathcal{A}, \mathcal{O}) \leftarrow \text{Adv}$

Exp_0 : does nothing

Exp_1 : measure $\mathcal{A}$ at location idx

$(\mathcal{A}, \mathcal{O}) \rightarrow \text{Adv}$

$\Pr[\text{Adv distinguishes } \text{Exp}_0 \text{ and } \text{Exp}_1] \leq \epsilon_{\text{VCCollapse}}^*$

Recap

[Kilian92]

Kilian's protocol:
 $\text{PCP} + \text{VC} \rightarrow \text{ARG}$

Classical Security

[CDGSY24]

Post-quantum Security

[CMSZ21]

Prior work

.....
This work

[BCS16;CDGS23]

IBCS protocol:
 $\text{IOP} + \text{VC} \rightarrow \text{ARG}$

Classical Security

[CDGS23]

Post-quantum Security

$\Rightarrow$ **Best** post-quantum secure succinct arguments
in the standard model (no oracles)



<https://eprint.iacr.org/2025/947>



Ziyi

Thank you!

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